

ON FREE VIBRATION OF STEEL PLATE EMBEDDED IN CONCRETE

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BY
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CERTIFICATE

This is to certify that the dissertation entitled "On Free vibration of steel Plate Embedded in concrete" by Sri Ravi Kant Bajpai in partial fulfilment of the requirements for the degree of M.Tech., has been carried out under my supervision and guidance. The matter embodied in this thesis has not been submitted elsewhere for a degree.



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ABSTRACT

Analytical study has been performed on free-vibration of steel embedded plate-anchor assembly which is commonly used to support pipelines ,machinery ,equipments ,etc. from R.C.C. members.

Finite element technique has been used to analyse the idealised model of the embedded plate-anchor assemblies for different combinations of plate thicknesses ,base stiffnesses ,and anchor stiffnesses .Two basic types of models were adopted based on the expected behaviours of the concrete base and anchors at the plate-concrete interface .

The results obtained from the different parametric variations of the plate-anchor assembly have been used to study the effects of such parameters ,viz.,plate thickness ,concrete base stiffness ,and anchor stiffness on the free-vibration characteristics of such plates .

ACKNOWLEDGMENT

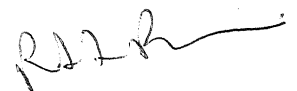
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Date 19-11-90



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LIST OF SYMBOLS

A_c	Tributary area of the base .
a	Length of the plate .
b	Width of the plate .
D	Flexural rigidity of the plate .
E_c	Modulus of elasticity of concrete .
E_s	Modulus of elasticity of steel .
f_c	Contact pressure .
f_{ck}	Characteristic strength of the concrete .
K	Modulus of subgrade reaction .
K_c	Stiffness of the concrete .
K_{as}	Stiffness of the anchors in shear .
K_{at}	Stiffness of the anchors in tension .
L	Effective depth of the concrete member .
U_a	Response of the plate .
X	Co-ordinate along X-axis .
Y	Co-ordinate along Y-axis .
γ	An exponent parameter .
ϕ	Flexibility coefficient .

CHAPTER 1

INTRODUCTION

1.1 GENERAL :-

Use of cast-in-place embedded steel plates in R.C.C. structures is very common in modern civil engineering constructions, e.g. Nuclear Power plants, Thermal Power stations, Steel Plants etc. These plates are generally provided to support machines, equipments, pipelines carrying running fluids etc. from the main R.C.C. members. Some typical embedded plates have been schematically shown in Fig 1.1.

The plate anchor assemblies are installed at the desired locations by attaching them suitably to the concrete formwork prior to actual concreting operation. Generally, steel rod welded to the plates are used as anchor as shown in Fig 1.1. In addition to the static loads and forces such support systems may also be subjected to machine vibrations, seismic inertias, and fluid dynamic forces (water, steam hammers) or combinations thereof.

In a great body of design problems, specifications ensuring that the plates or any systems will withstand applied static loads, prove to be inadequate. Rather, the designer must be concerned with the possibility that large cyclic displacements and stresses may be induced due to periodic or random time varying

forces acting on the system , or plates . It is well known that there exists a large number of discrete frequencies at which a plate may , undergo vibration by sustained time-varying forces of matching frequencies . These are said to be resonant , natural , or free vibration frequencies of the plate system . It is also known that associated with each plate natural frequency there is a distinct characteristic or mode shape which the system acquires as it vibrates .

It is apparent that the ability to conduct an accurate free vibration analysis of rectangular plate-anchor assembly system is absolutely essential if the designer is to be concerned with the possible resonance conditinos occuring with plate-anchor assembly due to the action of driving force system. In fact, free vibration of plate-anchor assembly (or any system) is an essential first step towards obtaining solutions for the forced vibration of plate-anchor assembly embdded in R.C.C. members .

1.2 ABOUT THE EMBEDDED PLATE-ANCHOR SYSTEM :-

Embedded plate-anchor system consists of a steel plate supported by the concrete base and attached to the base with anchors . Generally, anchor-plate assembly is cast-in-place type .Anchors are welded to the plate surface . This assembly is inserted in the R.C.C.member as described earlier . A steel member e.g. I section , T section , or channel section is welded to the plate surface to support the pipes , machines ,or working plate forms etc . The plate assembly thus becomes, in it s most regular form , a structure consisting of plate with it s central portion

~

stiffened due to welded steel members (see Fig 1.1) while resting on an elastic concrete base and attached to base at certain location with steel anchors. A typical plate-assembly is illustrated in Fig 1.2 .

1.3 LITERATURE SURVEY :-

Although the free vibration analysis of plates resting on an elastic foundation has received the attention of researchers for several centuries , its treatment , it seems , has left much to be desired . Amongst the literature cited no one seems to have hit the problem , as chosen , directly but provides certain background to the problem .

Basci , et.al.[1] have developed more accurate procedures in the generation of the stiffness and mass matrices of a thin rectangular plate which can be subsequently embedded in a usual finite element program . They considered a plate in a free-free state along its boundaries . The displacement functions for both the static and free-vibrational cases are derived based on a Levy type solution method for plate analysis. In Levy's method it is required that two opposite edges of the plate be simply supported, and it is assumed that the plate is infinitely long in the other direction . Furthermore, it is required that the lateral loading have the same distribution pattern in each section parallel to the longitudinal axis of the plate .Stiffness matrix for beams on elastic foundation was determine by Eisenberger and Yankelevsky [3]. According to them a single element is required to exactly represent a continuous part of a beam on a Winkler foundation .

Laura and Gutierrez [4] dealt with the determination of the fundamental frequency of vibration of circular and regular polygonal plates elastically supported over a non-homogeneous foundation as shown in Fig 1.3. Both cases are tackled in a uniform fashion by adopting polynomial co-ordinate function [4] given as

$$U_a = A1 (X^\gamma + \alpha_3 X^3 + \alpha_2 X^2 + \alpha_1 X) (Y^\gamma + \beta_3 Y^3 + \beta_2 Y^2 + \beta_1 Y)$$

Where,

U_a = Response of the plate .

$A1$ = A Constant .

γ = An exponential parameter .

$$\left\{ \begin{matrix} \alpha_1, \alpha_2, \alpha_3 \\ \beta_1, \beta_2, \beta_3 \end{matrix} \right\} = f(\phi_1', \phi_2', \phi_3', \phi_4', \gamma)$$

$$\phi_1' = \frac{\phi_1' D}{a}, \quad i = 1, 2$$

$$\phi_1' = \frac{\phi_1' D}{b}, \quad i = 3, 4$$

a, b = Sides of the rectangular plate .

D = Flexural rigidity of the plate .

ϕ = Flexibility coefficient [4]

$$\bar{X} = \frac{X}{a}$$

\bar{X} = Co-ordinate along X-axis .

$$\bar{Y} = \frac{Y}{b}$$

\bar{Y} = Co-ordinate along Y-axis .

Finally, using the Ritz method they obtained the frequency. They considered the following boundary conditions in their study.

Circular plate :

The plate periphery is simply supported with elastic restraint against rotation.

Regular Polygonal Plates :

- (i) The plate is simply supported along its periphery
- (ii) The plate is clamped along its periphery .

Laura and Gutierrez performed another study [5] regarding the forced vibration of rectangular plates embedded in a non-homogeneous foundation as illustrated in Fig 1.4 . The plate dynamic amplitude , in this case , is approximated by means of a polynomial co-ordinate function as given by the equation (1) above which contains an undetermined parameter γ as an exponent . The governing functional is minimized with respect to the unknown coefficient (A1) of the co-ordinate function and with respect to exponential parameter (γ).

Making the sinusoidal force zero the given problem in this paper results in free vibration of rectangular plate embedded in a non-homogeneous foundation . The proposed procedure is simple and apparently yields first order approximation which may be useful for design purposes .

Volos and Dovganich [6] studied the problem of plate resting on elastic foundation including the effects of shear deformation . They considered a rectangular transversely isotropic

(same properties in the transverse direction) plate of constant thickness lying on an elastic base . The problem was solved by the method of approximate separation of variable .Natural frequencies and the corresponding vibrational modes of plates were investigated .

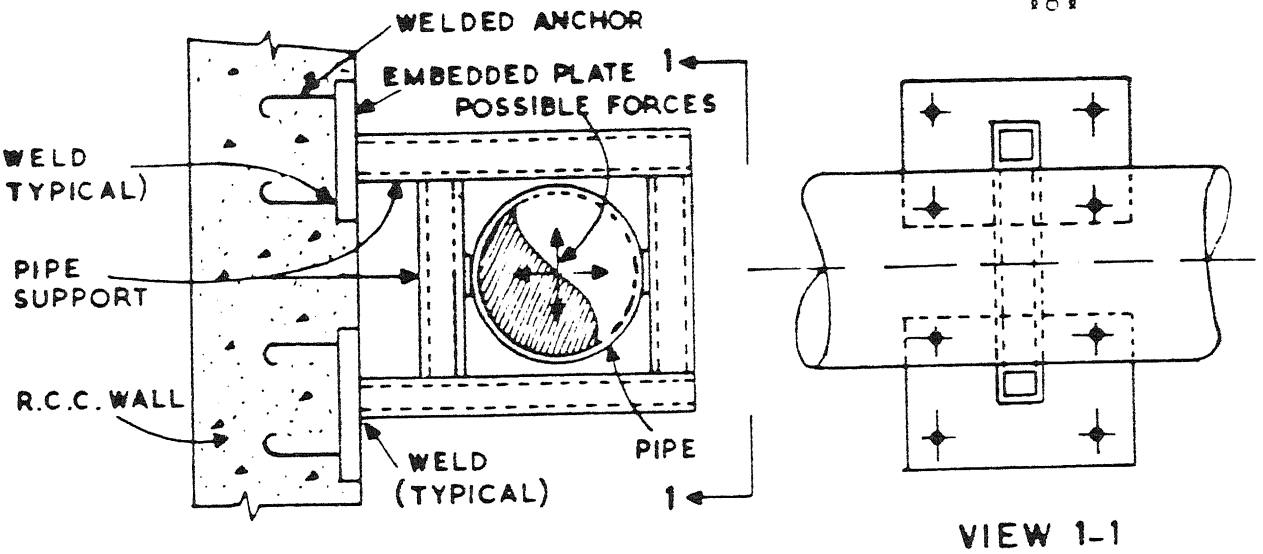
1.4 OBJECT AND SCOPE OF THE PRESENT INVESTIGATION :

The current trend towards the analysis and design of such embedded plates is to make simplifying assumptions regarding the behaviour of the plate-anchor system . A series of assumptions are generally made in order to simplify the complexity of the problem. The objective of the present study is to study analytically the effect of parametric variation of the plate-anchor assembly, viz., plate thickness, concrete base stiffness, and anchor stiffness on the free-vibration characteristic of such plates.

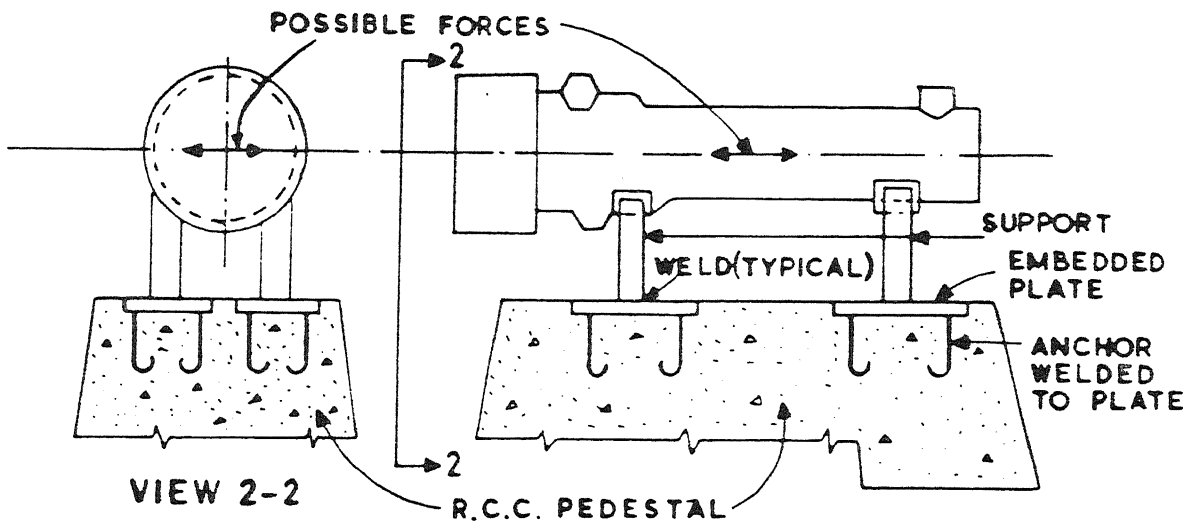
The behaviour of the embedded plate-anchorage system is dependent on several factors such as the plate flexibility , the anchor stiffness and the concrete base stiffness. A typical embedded plate-anchor assembly has been analysed to study the effects of variations of plate thickness, concrete base stiffness, and anchor stiffness on the free-vibration characteristics of such plates.

The load-deflection behaviour of the anchors and concrete base have been assumed to be linear . The concrete base has been assumed to consist of a series of linear springs which do not carry any shear stresses (i.e. Winkler type).The base plate has

been analyzed within the limits of the thin plate bending theory .



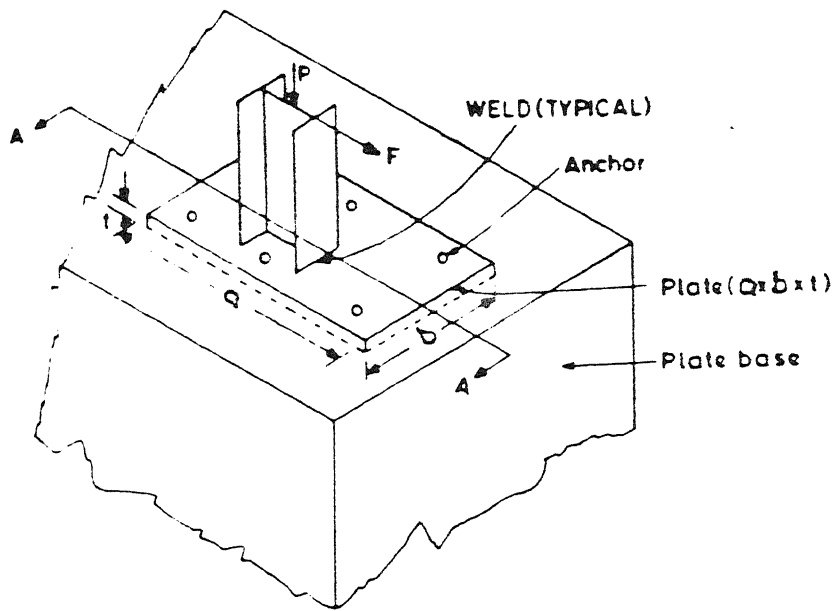
A. EMBEDDED PLATES FOR PIPE SUPPORTS



B. EMBEDDED PLATES FOR VESSEL SUPPORTS

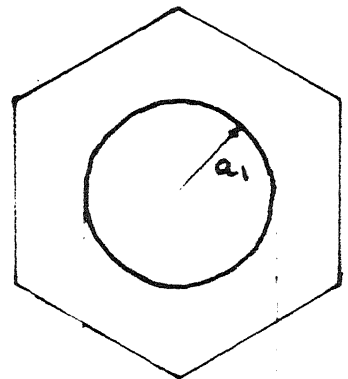
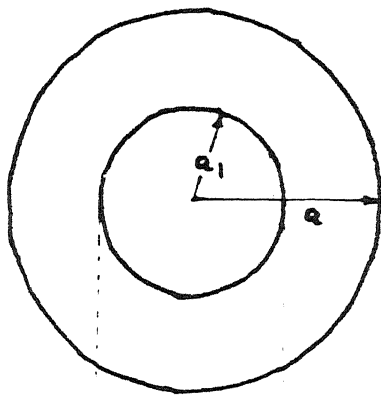
SOME TYPICAL CAST-IN-PLACE STEEL EMBEDDED PLATES.

FIG. 1.1

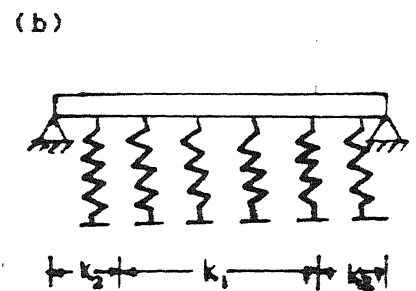
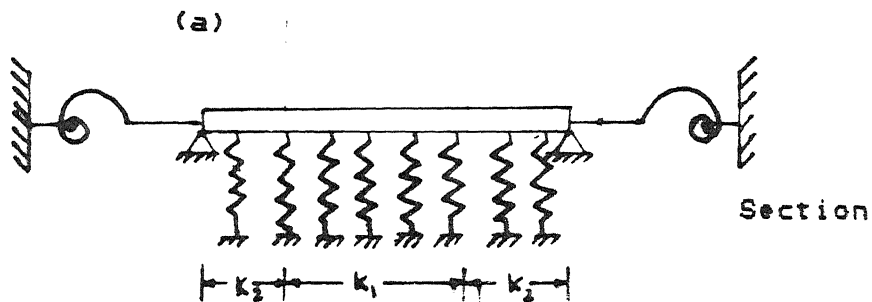


EMBEDDED PLATE ASSEMBLY

FIG.1.2



Plan

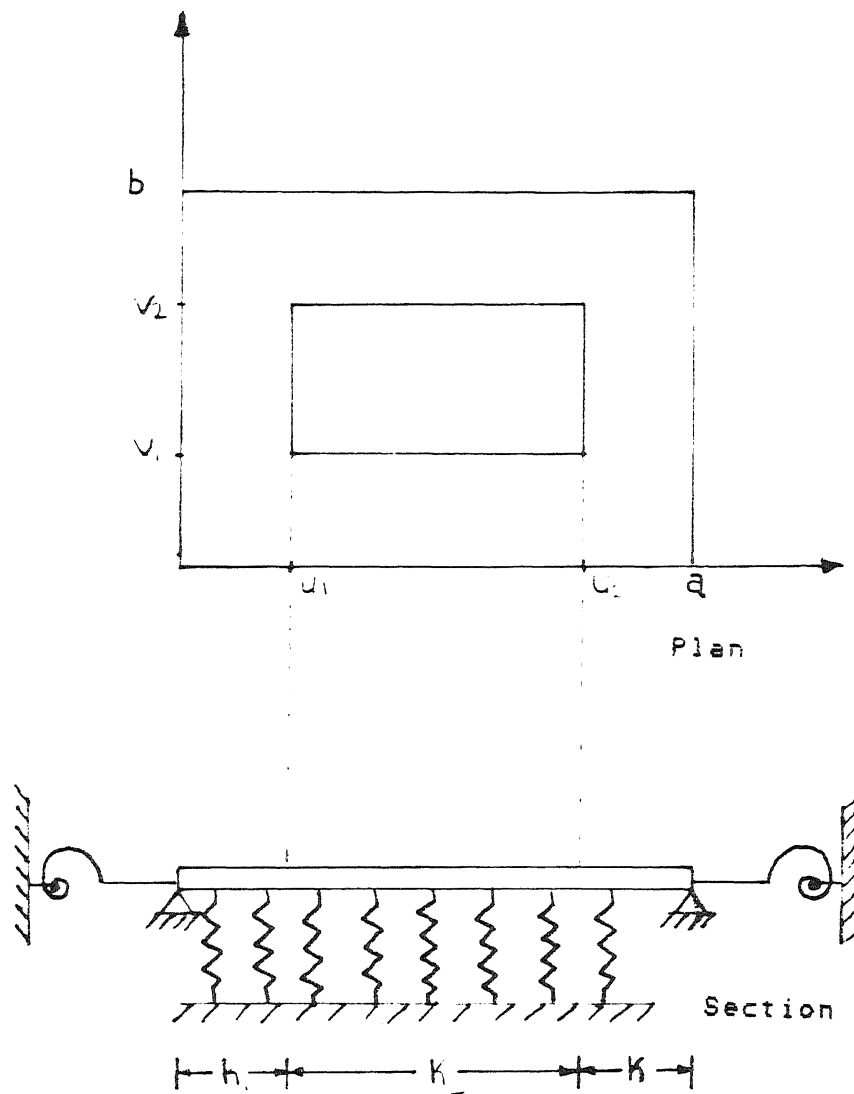


Section

(a). CIRCULAR PLATE ELASTICALLY RESTRAINED AGAINST ROTATION ALONG THE EDGE .

(b). PLATE OF REGULAR POLYGONAL SHAPE .

FIG. 13



VIBRATING RECTANGULAR PLATE EMBEDDED IN A NON-HOMOGENEOUS FOUNDATION .

FIG. 14

CHAPTER 2

2.1 DEVELOPMENT AND ANALYSIS OF THE MODEL :-

A typical embedded plate assembly as shown in Fig 2.1 has been selected for the present study . The assembly consists of three basic components , the plate , the anchors , and the concrete base . Each of these components has been discussed below in connection with the free vibration study of the system

2.1.1 THE PLATE PROPER :-

The free-vibration characteristic of the embedded anchor-plate system are influenced partly by the following factors attributed to the plate proper :

- (i) Thickness of the plate relative to its other dimensions.
- (ii) Aspect ratio of the plate.
- (iii) Properties of the plate material.

The system has been analyzed by finite element method with the following assumptions regarding the plate behaviour

- (i) The plate is thin , and therefore , thin plate theory is applicable
- (ii) Vibrations are of small amplitude .
- (iii) All the assumptions of thin plate theory for small deflection of the plate are valid .
- (iv) The central portion of the plate where the steel member is attached has been assumed to be rigid .

- (v) Plate material is homogeneous , isotropic , and linearly elastic .

2.1.2 THE ANCHORS :-

Embedded anchor exhibit non-linear load-deflection behaviour when subjected to pull-out and/or shearing forces . The exact behaviours depend upon the shape and size of the anchor , strength of the concrete in which anchor is embedded , and the bounding resistance developed between concrete and anchor . The load-deflection behaviours obtained from the experimental investigation for j-shaped anchors embedded in concrete masonry are illustrated in Fig 2.2 [7]. It can be observed from Fig 2.2 that the load-deflection curve for a typical anchor is initially linear and becomes increasingly non linear with the increase of load . In the present study the load-deflection behaviour has been assumed to be linear in order to calculate the anchor stiffness in tension . On the basis of the experimental results obtained by Brown [7] the stiffness in shear of the anchor has been assumed to be one-half of the stiffness in tension .

In the present work ,the anchors have been considered to be active in compression , as well . In modelling the anchor stiffness in compression has been assumed to be of the same magnitude as in tension . The embedded plates have been analysed for the following two cases with the view to study the free vibration behaviour while modelling the anchors :

- (i) Considering the anchors to be active in tension only .
- (ii) Considering the anchors to be active both in tension and in compression .

2.1.3 THE BASE :-

The response of the concrete base forms a topic of fundamental importance in the analysis of plate-base interaction problems . Prediction of the response of the concrete can be made basically from the knowledge of the complete stress-strain characteristics of the plate-base .Considering the varieties of the plate-base which are normally encountered in engineering practice , it appears unlikely to obtain a generalised stress-strain relation to model such bases .

The inherent complexity in the behaviour of the real concrete base , has led to the development of many idealised models of the base behaviour , especially , for the analysis of the plate-base interaction problem . The three basic types of models which can be used to represent the response of the concrete base are discussed below .

2.2 THE WINKLER MODEL [8] :-

The elastic base can be assumed to consist of mutually independent spring elements , capable of resisting compressive forces only . The deflection at any point on the contact surface is directly proportional to the contact pressure at that point , and independent of contact pressure at other point : that is

$$F_c = K Z$$

Where 'K' is the modulus of subgrade reaction having dimension of pressure per unit depth , F_c is the contact pressure , and Z is the deflection of the plate . One important feature of this model is that the part of the base out side the contact area

2.3 THE ELASTIC SOLID MODEL [8] :-

It is common experience that surface deflections occur not only immediately under the contact region , but also within certain limited zone outside the contact region . To account for this behaviour ,media have been idealized as three dimensional elastic half-space . Thus the problem can be solved by using the theory of classical elasticity .

2.4 TWO PARAMETER MODEL [8] :-

The deficiency of Winkler model in accounting for base area beyond the contact region and the complexity of mathematical solution of an elastic-solid model can be avoided by considering two independent elastic constants . Several such models have been developed by researchers .These models have been discussed in brief by Selvadurai [8].

The mathematical complexity involved in solving a two-parameter model and an elastic-solid model have led to the adoption of the Winkler model for representing base response . The concrete base of Fig 2.1 has been idealized on the lines of Winkler model with the following assumptions :

- (i) The base consists of mutually linear elastic springs capable of resisting compression only .
- (ii) All the springs shall be of the same stiffness if placed at the nodal points of a two dimensional grid consisting of equally spaced orthogonal lines irrespective of the

geometry . This is a modification of the original Winkler model .

2.5 THE BASE-PLATE INTERFACE :-

The base-plate interface is assumed to be smooth and frictionless in the region of contact . It is assumed that shearing force , if any , shall be resisted by the anchors only .

After applying all the assumptions on the behaviour of each component of the plate assembly , the problem reduces to the solution of a linear- elastic thin plate, resting on a smooth , tensionless and friction less , linear elastic media and attached to the linear elastic tension/compression / shear springs (anchors) at specified locations . The linear-elastic media of concrete base can be represented by linear-elastic mutually independent compression springs .

2.6 THE FINITE ELEMENT ANALYSIS :-

2.6.1 GENERAL :-

The model as described earlier shall be analyzed by varying the three parameters , namely : the plate thickness , the base stiffness (compression spring stiffness) and the anchor stiffness (spring stiffness with two cases as given in section 2.1) Using the finite element technique , frequencies and corresponding eigenvectors (mode shapes) are determined for different cases .

In order to analyze the plate-anchorage system a finite

element model was developed . The base plate was idealized by a mesh of square and rectangular plate elements connected at the corners of the nodal points . The concrete is represented by the spring elements attached to the nodal points and to another fixed point . Since ,the concrete elements can not resist the tension , the nodal points are free to translate in the +Z direction (vertically up) .The one dimensional boundary elements with the axial stiffness was used to represent these elements .

The same element type was used to represent the anchors for two cases ; Anchors resisting tension only and anchors resisting both tension and compression . A compression-only element was also placed at each anchor location connecting the same two nodal points in order to represent the concrete base element .The shear restraint of the anchors was represented by elements lying in the plane of the plate , with a stiffness (half of the tension stiffness) representing the shear stiffness of the anchors . These elements are active in tension as well as in compression

The SAP IV computer programme [12]was used to analyze the model . The thin plate element available in the programme is a quadrilateral of arbitrary geometry formed from four compatible triangles . The central node is located at the average of the co-ordinates of four corner nodes . The element has six interior degrees of freedom which are eliminated at the element level prior to assembly . Therefore , the resulting quadrilateral has twenty four degrees of freedom i.e. six degrees of freedom per node in the global co-ordinate system .In the analysis of flat plates the

to the plate surface θ_z is not defined . Therefore, the rotation normal degree of freedom must not be included in the analysis, and hence, rotational normal degree of freedom (θ_z) has been restrained in the analysis .

Symmetry was exploited to reduce the computer time and cost . The model used for analysis is symmetric about X and Y axes passing through the centre of the plate . Therefore , only quarter right upper part (See Fig.1.2, Fig 2.3 and Fig 2.4) was analyzed by imposing conditions of symmetry . The conditions of symmetry as imposed on the nodes located on the axes of symmetry of the entire plate are as follows .

- (i) Along X-axis : Rotation about X axis : $\theta_x = 0$
Deflection in Y-direction : $V = 0$
- (ii) Along Y-axis : Rotation about Y axis : $\theta_y = 0$
Deflection in X-direction : $U = 0$
- (iii) Stiffness of the concrete or anchor elements situated in the axes of symmetry is equal to half of the stiffness of the concrete or anchor element situated elsewhere .

The finite element grids , the node numbering , the element numbering ,and element-arrangements are shown in Fig 2.3 and Fig 2.4 for two typical finite element models , which were analyzed .

In SAP IV the solution for frequency and mode shapes (eigenvectors) is sought by the two techniques ; determinant search method , and subspace iteration method as described in the SAP IV manual [12]. Further relevant details about these

The programme performs the solution for frequency and modes shapes using either of the above two distinct algorithms . Determinant search algorithm requires that the upper triangular band of system stiffness matrix fits into high speed memory (core) which corresponds to one equation block .

The subspace iteration algorithm is used if only portions (fractions) of the system matrix can be retained in core ;i.e. the matrix (even though in band form) must be manipulated in blocks .

The programme will automatically select the subspace iteration procedure for eigenvalue solution if the model is too large for the in-core algorithm .

2.7 ITERATIVE SCHEME FOR DETERMINATION OF FUNDAMENTAL MODE AND FREQUENCY :

As discussed before the concrete elements have been defined as compression-only elements whereas the anchor elements have been defined ; firstly, as tension-only elements and secondly, as tension/compression elements . For the first case ,the solution procedure requires that the concrete elements having tension and anchor tension elements having compression be identified after the first run of SAP , and subsequently, after each iterative run as needed . This is done by modifying the SAP so as to notify those nodes which are lifted-off and the anchor node undergoing compression . The SAP is re-run at each iterative step after eliminating such spring elements from the input . This procedure shall be repeated till all the concrete elements come

tension .

For the second case, the solution procedure requires that the concrete elements having tension be only identified after first run of SAP, and subsequently after each iterative run as needed, keeping the anchor spring elements always active. As discussed above in the last paragraph the procedure is continued till all the concrete elements come under compression.

In order to implement the above iterative scheme the following three external programmes were incorporated :

- (i) Programme to modify the input data at each iterative step for the analysis case when the anchor elements have been defined as tension only elements .
- (ii) Programme to modify the input data at each iterative step for the case when anchor elements have been defined as tension/compression elements .
- (iii) Programme to check the convergence :

The purpose of this programme is to check that the same nodes which were lifted in the $(i-1)^{th}$ iteration were also lifted in the i^{th} iteration when the i^{th} iteration becomes the converged step .The flowchart for the iterative scheme is given in the Fig 2.5 and Fig 2.6

2.8 SELECTION OF NUMERICAL VALUES AND PARAMETERS :-

2.8.1 EMBEDDED PLATE :

- (i) Thickness of the plate : Plates of thicknesses 16 mm to 40 mm are generally used in industrial projects for use in anchor

used as they become too flexible to be used as moment resistant plates . Also , plates of thicknesses more than 40 mm are generally not required for common types of applications .

In present study commonly used thicknesses of embedded plates e.g. 16 , 20 , 24 , 28 , 32 , and 40 mm have been selected . The plate material has been considered to ^Eof standard quality as per IS : 226-1975 [9].

- (ii) Density of the plate material : 7.85 t/m^3 [14]
- (iii) Elastic modulus of plate material : $2 \times 10^5 \text{ n/mm}^2$ [14]
- (iv) Poisson ratio of plate material : 0.3 [14]

2.8.2 ANCHOR STIFFNESS :-

Experimentally obtained load-deflection data on j-shaped anchors embedded in grouted concrete masonry are given in Fig 2.2 [7] . The linear values of the stiffnesses have been derived from Fig 2.2 for use in the present study . The values of anchor stiffness $1 \times 10^4 \text{ kn/m}$, $2 \times 10^4 \text{ kn/m}$, $4 \times 10^4 \text{ kn/m}$, and $8 \times 10^4 \text{ kn/m}$ correspond to the anchors of the sizes 10mm , 12mm , 25mm , and 32mm diameters , respectively . In the present study, anchors of diameters 25mm and 32mm were selected . The stiffness of the anchors in shear was found to be about half of the stiffnesses in tension , when subjected to combined loading [7]. Therefore , shear stiffnesses of anchors have been kept equal to half of the stiffness in tension throughout .

2.8.3 BASE STIFFNESS :-

An approximate method of estimating the values of concrete base stiffnesses was using the tributary area method as suggested by Diluna et.al.[10] was adopted in the present investigation . The method is explained below :

$$K_c = \frac{A_c \cdot E_c}{L}$$

K_c = Concrete Base stiffness .

A_c = Tributary area of concrete base .

= 1.0 m² for per unit area calculation .

L = Effective depth of base (may be considered approximately equal to 0.6 m. for a common type of R.C.C. member .

E_c = Modulus of elasticity of concrete .

= 5700 / $\sqrt{f_{ck}}$ as per IS: 456-1978 [11]

Where, f_{ck} = Characteristic strength of concrete

= 20 N/mm² for M₂₀ concrete .

Therefore for M₂₀ concrete

E_c = 2.55 × 10⁴ N/mm², and hence,

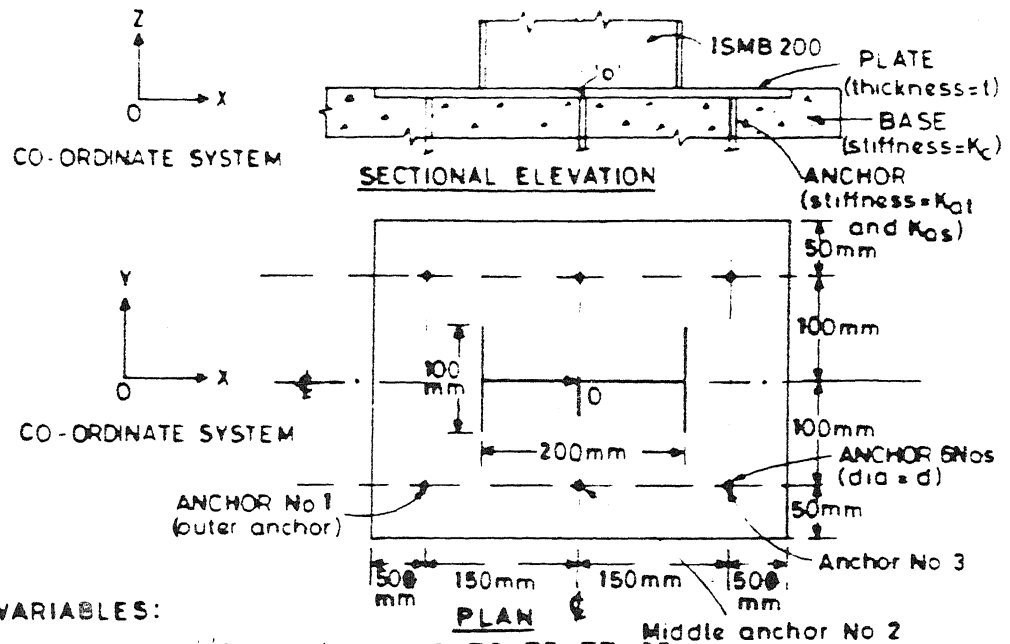
K_c = 4.0 × 10⁷ KN/m²/m , for

A_c = 1.0 m² and L = 0.6 m

It is felt that the actual value of K_c will be much higher than that obtained by the tributary area method due to the confining effect of the surrounding concrete mass also in certain cases the values of ' K_c ' can be lower due to the lesser thickness of the R.C.C. members than the assumed value . Keeping this in view ,the following values of ' K_c ' were considered in order to broaden the applicability of the results of the present investigation :

(i). K_c = 4.0 × 10⁶ KN/m²/m (Flexible Base)

- (ii). $K_c = 2.0 \times 10^7 \text{ KN/m}^2/\text{m}$ (Moderately Flexible Base)
- (iii). $K_c = 4.0 \times 10^7 \text{ KN/m}^2/\text{m}$ (As per Tributary Area
Method).
- (iv). $K_c = 2.0 \times 10^8 \text{ KN/m}^2/\text{m}$ (Moderately Rigid)
- (v). $K_c = 4.0 \times 10^8 \text{ KN/m}^2/\text{m}$ (Rigid Base)



VARIABLES:

Plate thickness t (in mm) = 16, 20, 24, 28, 32, 40

Anchor diameter d (in mm) = 25, 32

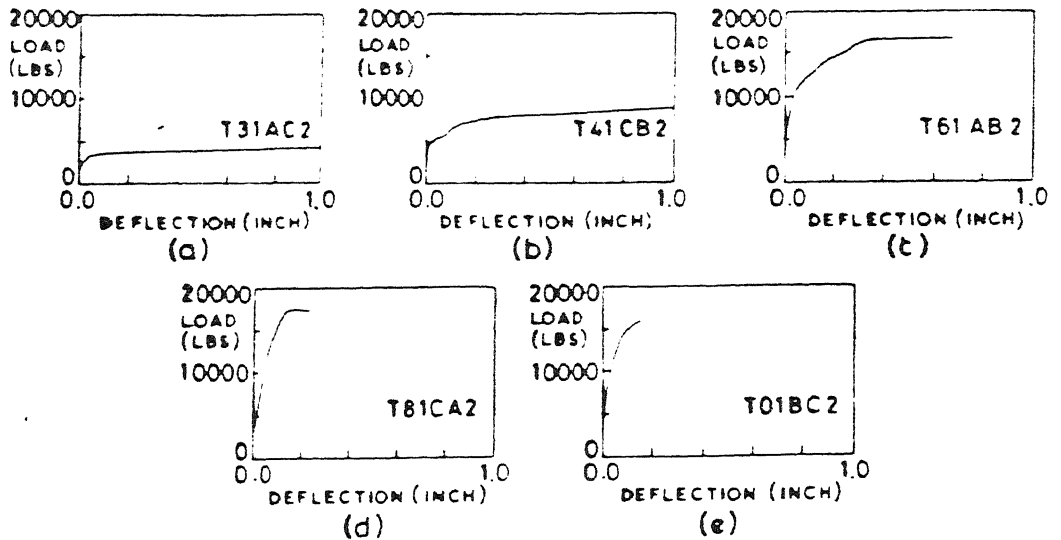
Anchor stiffness in tension K_{at} (in 10^4 KN/m) = 4, 8

Anchor stiffness in shear $K_{as} = 1/2 K_{at}$

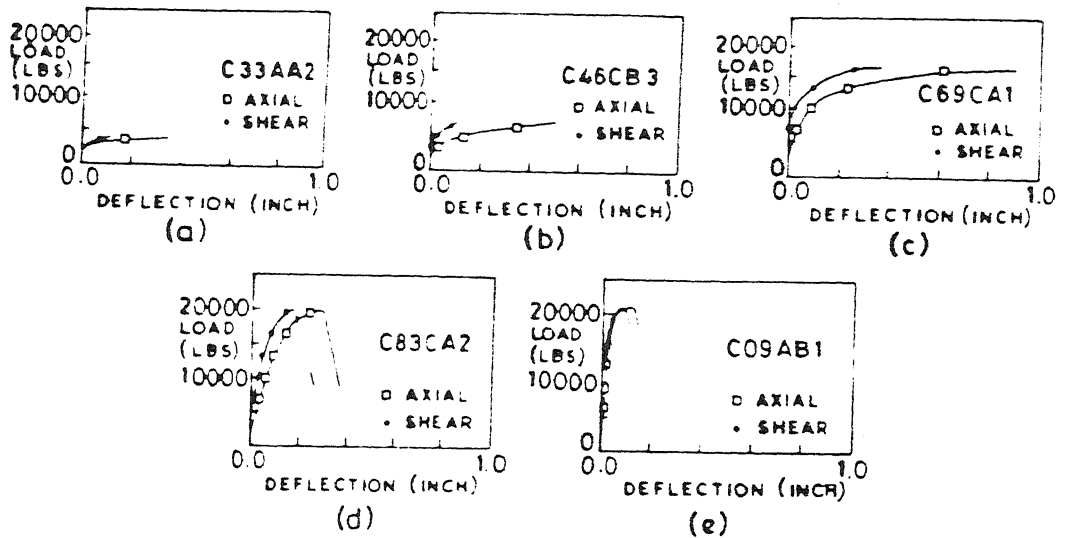
Base stiffness (in 4×10^6 KN/m²/m) = 1, 5, 10, 50, 100

TYPICAL PLATE ANCHOR ASSEMBLY.

FIG. 2.1



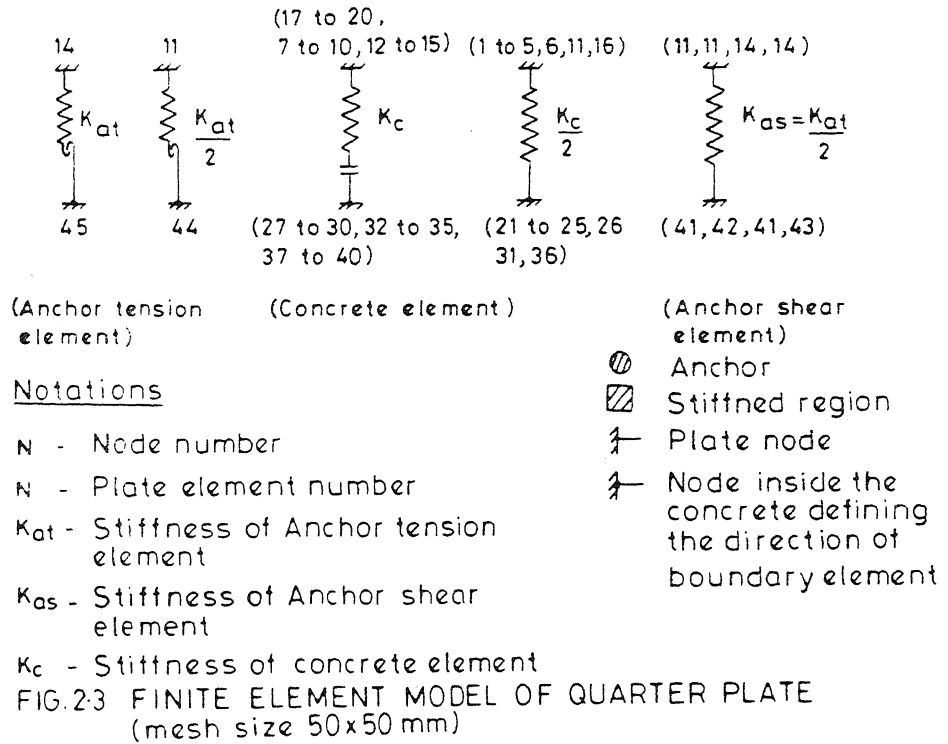
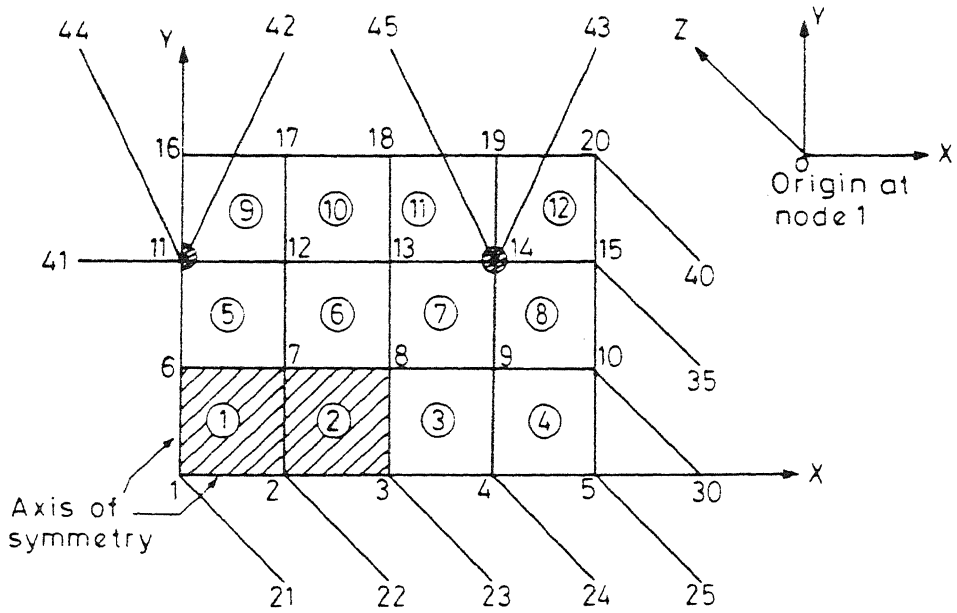
TENSION TEST: (a) 3/8 in. Bolt, (b) 1/2 in. Bolt, (c) 3/4 in. Bolt, (d) 1 in. Bolt, (e) 1-1/4 in. Bolt.



COMBINED TEST: (a) 3/8 in Bolt, (b) 1/2 in Bolt, (c) 3/4 in Bolt, (d) 1 in Bolt, (e) 1-1/4 in Bolt.

FIG. 2.2

EXPERIMENTAL DATA FOR J-SHAPED ANCHORS IN CONCRETE MASONRY.



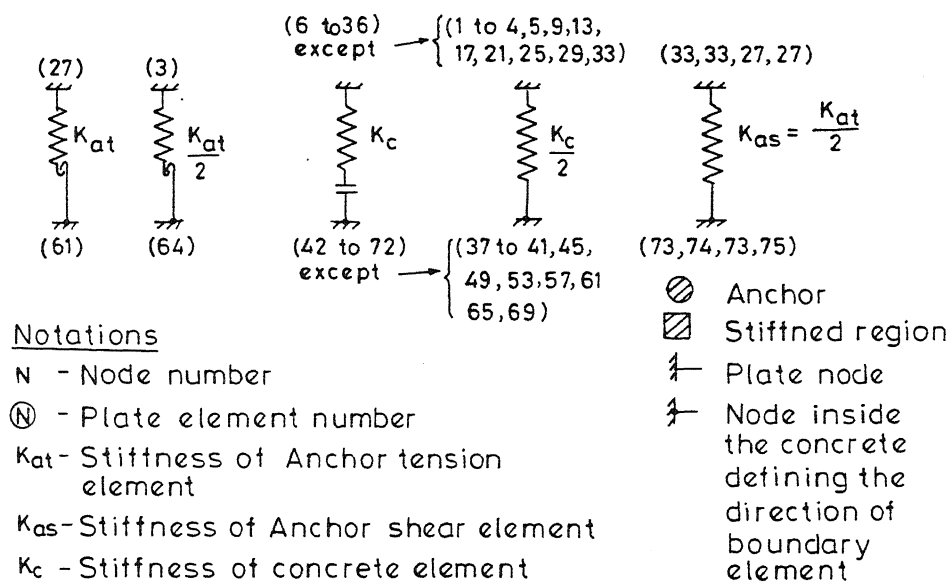
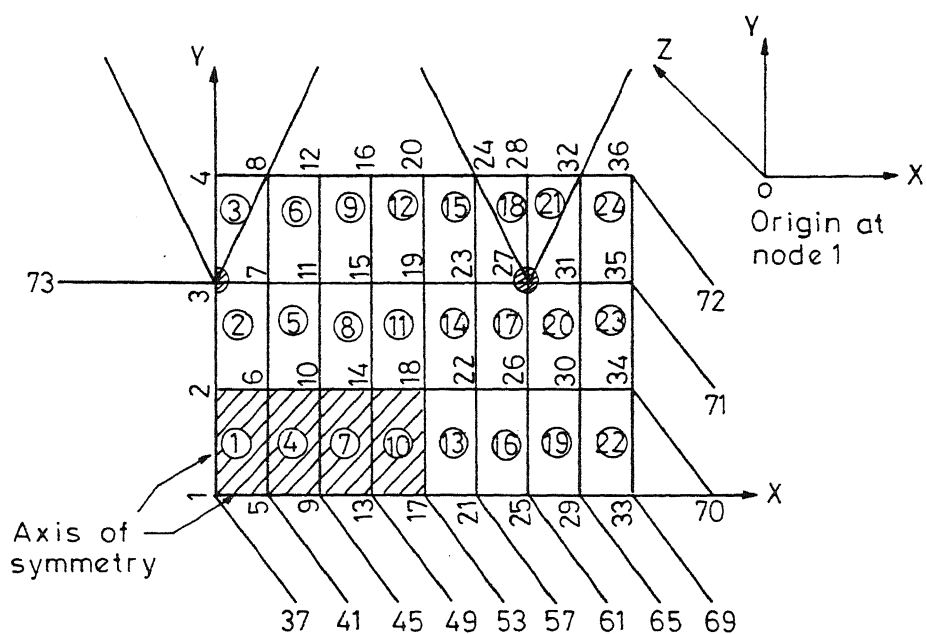


FIG.2.4 FINITE ELEMENT MODEL OF QUARTER PLATE
(mesh size 50 x 25 mm)

FLOW CHART OF THE ITERATIVE SCHEME

Model Type I : (Anchor active in tension only)

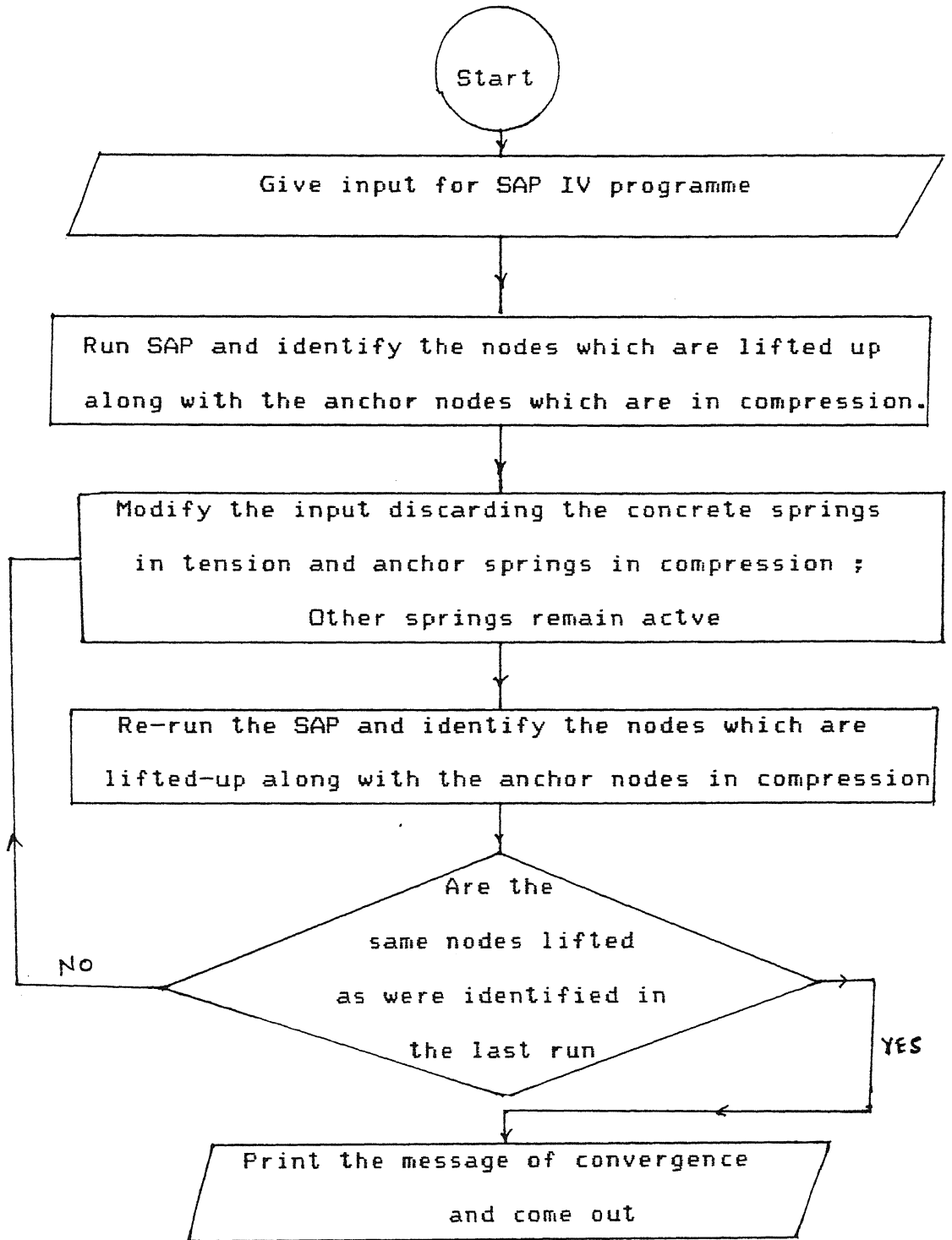


FIG. 2.5

Model Type II : (Anchor active both in tension and in compression)

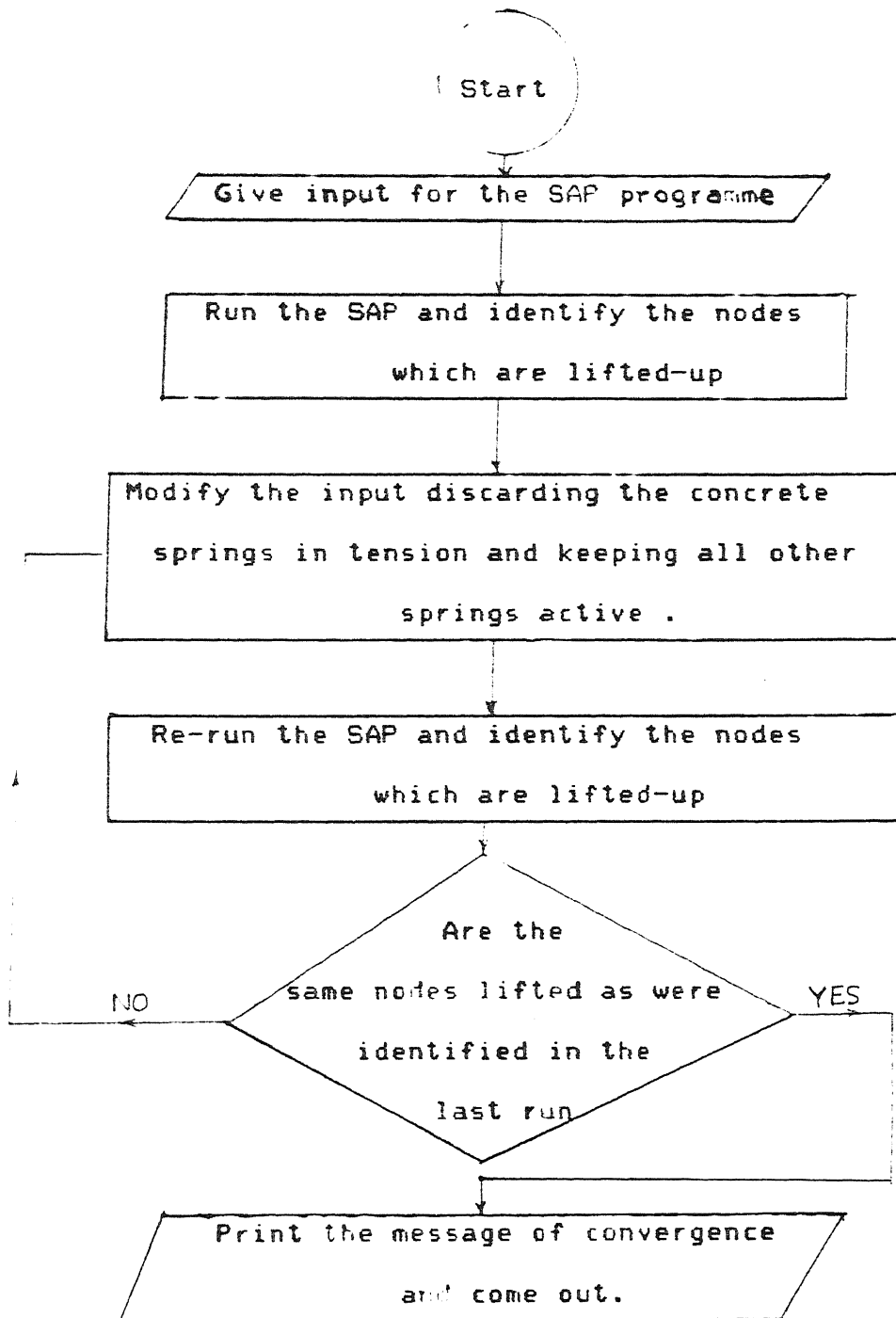


FIG. 2.6

CHAPTER 3

RESULTS AND DISCUSSION

3.1 GENERAL :-

Total of 240 cases were analysed based on the following Models :-

Model type I : Considering the anchor to be active in tension only.

Model Type II : Considering the anchor to be active both in tension and compression .

The analysis results have been presented through graphs and tables for different combinations of plate thicknesses, anchor stiffnesses, and base stiffnesses .Numerical computations were all carried out using SAP IV programme (refer discussion in chapter 2) on the H.P.9000-850 system in the computer center, I.I.T. Kanpur .

Three mode shape patterns of the embedded plate assembly were finally observed after analyzing the 240 cases for different combinations of plate thicknesses, anchor stiffnesses, and base stiffnesses . Each of these patterns has been discussed in the following .The SAP IV programme outputs the frequency and the corresponding eigenvectors for each case of embedded plate assembly along with the method of computation adopted .

The mode shape pattern of the plate assembly as illustrated in Figs.3.1 through 3.10 are described below .

Figs. 3.1-3.5 :- These figures show how the plate converged to the final mode shape for the Model Type I .

After the first iteration it has been observed (Fig.3.1) that a large portion of the plate was lifted excepting some area near the upper right-hand corner .

After the second iteration (Fig.3.2) it has been observed that all the plate area was in contact excepting some localized region near the upper right-hand corner appearing lifted-off .

The third iteration (Fig.3.3) produced similar pattern of the plate mode shape as was observed after second iteration but with smaller area in contact and large area under lift-off relative to the last iteration .

The fourth iteration (Fig.3.4) also produced similar mode shape pattern with the indication of the spread of lift-off region around the upper right-hand corner .

Finally, the fifth iteration (Fig.3.5) gave the convergence with exactly the same mode shape pattern as the last one, and thus producing the converged frequency and the corresponding eigenvector .

The mode shape patterns illustrated in Figs.3.1-3.5 are representative of the plate behaviour through the process of

fundamental frequency .It has been observed from the results (Tables 3.1-3.10) that the X and Y translational components in the plane of the plate, are negligibly small compared to those in Z-direction (vertical direction). The plate frequency extracted after the first iteration appeared to be maximum as all the springs (concrete and anchors) were considered active .The subsequent iterations produced frequencies having magnitudes always less than that of the first iteration ,although their mutual values were observed to be fluctuating depending on the inclusion of the numbers/distribution of active springs .

Figs. 3.6-3.10 : These Figures show how the plate converged to the final mode shape for Model Type II .

On studying these Figures it has been observed that mode shape pattern at each iteration step is similar to what has been observed in the Model Type I analysis as illustrated in Figs 3.6-3.10 although the frequency values have appeared to be different between two cases .

It is observed that excepting for the first iteration, the frequency value extracted at each subsequent iteration-step of Model Type II analysis has been found to be higher than the corresponding frequency value of Model type I analysis .This is due to the fact that in Model Type II the anchor springs were considered to be active in both compression (plate depression) and tension (plate lift-off) and thus adding to the system stiffness .The frequency-values for the first iteration in both the models were exactly the same due to the fact that all the

springs (concrete and anchors) were considered to be active before the first iteration in both the models .

Similar mode shape patterns at intermediate steps as well as converged step were observed when the plate assembly was analysed using the mesh size of 50x50 mms .This pattern of plate mode shape observed for the different combinations of the plate thicknesses , anchor diameters , and base stiffnesses are given in Table 3.2.1 for the two models and mesh sizes .

3.1.2 Mode shape pattern II : In this pattern the complete plate has been found to be in contact (i.e. no plate lift-off) as illustrated in Fig. 3.11 (Model Type I) and Fig 3.12 (Model Type II). This pattern of the plate mode shape was observed for certain number of combinations of plate thicknesses , anchor diameters , and base stiffnesses for the two models and mesh sizes as given in Tables 3.2.2 .

3.1.3 Mode shape pattern III : In this pattern of plate mode shape the complete plate was found to be under lift-off which has been illustrated in Fig 3.13 (Model Type I) and Fig. 3.14 (Model Type II) .This pattern of plate mode shape was observed for certain number of combinations of plate thicknesses , anchor diameters , and base stiffnesses as given in Table 3.2.3 .

3.2 VARIATION OF FREQUENCY WITH PLATE THICKNESS :-

In order to study the frequency of the embedded plate assembly , various graphs were made for the frequency with the variation of plate thickness and base stiffness keeping anchor

stiffness unaltered .This exercise was undertaken and completed for the two analysis models and two mesh sizes .All such graphs are given in Figs. 3.15 -3.18 .

Figs. 3.15 and 3.16 give the variation of the frequency of the plate-assembly with the variation in plate-thickness considering the three representative values of base stiffness ,as obtained from Model-I and Model-II analysis using the mesh size of 50x50 mms .

It has been observed from these Figs. 3.15 and 3.16 that the plate assembly frequency generally increases with the increase of plate thickness for all the representative base stiffness excepting in some special cases . It has also been observed that for a given plate thickness , frequency of the assembly generally increases with the increase of the base stiffness value . For the base stiffness value of $2 \times 10^7 \text{ KN/m}^2/\text{m}$ (half the value calculated based on tributary area method) and the plate thickness of 32 mm , the plate assembly mode shape gave a complete lift-off condition (Figs.3.13 and 3.14) (i.e.relatively flexible system) and thus a fall in plate assembly frequency .For the lowest base stiffness value of $4 \times 10^6 \text{ KN/m}^2/\text{m}$ as has been considered in this work,frequency reached the maximum value for the plate thickness of 32 mm ,beyond which a falling trend was observed in the magnitude of frequency .

Figs. 3.17 and 3.18 give the variation of frequency with the plate thickness using 50x25 mesh . These Figures demonstrate similar trend as discussed above for Figures 3.15 and 3.16 excepting the mode shape is completely a lift-off(Figs.3.13 and

3.14) one in case of 32mm thick plate-assembly for base stiffness value of $4 \times 10^6 \text{ Kn/m}^2/\text{m}$.

Tables 3.24 and 3.25 give the frequencies for different combinations of base stiffness for two different mesh sizes of 50x50 mms and 50x25 mms as obtained from Model Type I and Model Type II analysis respectively. It is seen that frequency increases with increase in plate thickness and concrete spring stiffness. This trend is not followed where the plate is completely lifted-up. Moreover the frequency as obtained from Model Type 2 analysis have been found to be higher than those obtained from Model Type I analysis.

It is also seen that for certain plate thicknesses and base stiffnesses frequency converges towards lower bound when mesh size is reduced from 50x50 mms to 50x25 mms. For base stiffness of $4 \times 10^8 \text{ kn/m}^2/\text{m}$ there is no convergence for this combination of plate thickness, base stiffness, and anchor diameter.

Tables 3.26 and 3.27 give the frequency for different combinations of plate thicknesses and base stiffnesses for two mesh sizes 50x50 mms and 50x25 mms as obtained from Model Type I and Model Type II analysis, respectively. Observations made in this case have been found to be similar to those made from Tables 3.24 and 3.25 above.

3.3 EIGENVECTORS BASED ON VERTICAL TRANSLATIONS ONLY :-

Tables 3.4 through 3.13 give only Z-translation element of eigenvectors for different combinations of plate thicknesses.

anchor diameters ,and concrete spring constants considering mesh size of 50x50 mms and Model Type I and Model Type II . Other elements of eigenvectors, such as X-translation and y-translation elements are not presented here ,as these elements have found to be of insignificantly small magnitudes .However other elements are all obtainable from the detailed computer outputs .

From these Tables , it can be observed that as thickness of the plate increases the behaviour of the plate tends towards rigid behaviour . This is evident from the nodal point deflections of the deformed plate (eigenvector) which appeared predominantly at the same level of deflection .This is also partly due to the relative stiffness of the plate compared to base stiffness . When the eigenvector of the plate assembly gives complete lifted-up condition ,plate becomes supported only by the anchor springs, and hence, the relative rigid behaviour of the plate becomes more pronounced .

Tables 3.14 through 3.23 give only z-translation for different combinations of plate thicknesses, anchor diameters ,and base stiffnesses considering mesh size of 50x25 mms for Model Type II .

It is observed that for this mesh size and pattern of the node numbering (Fig.2.4) ,no solution was obtained for plate thickness of 16 mm . A message "Rigid body mode found " was given by SAP IV while computing . The reason can be referred from SAP IV manual [11] where it states : " In computer arithmetic the element d_{nn} of the matrix D , in the triangular factorization of the stiffness matrix , i.e. $K = L D L^T$, is small

when compared with the other elements of the matrix D . If this condition occurs the program stops with above message ". Other observations are similar as described earlier for Tables 3.4 - 3.13 .

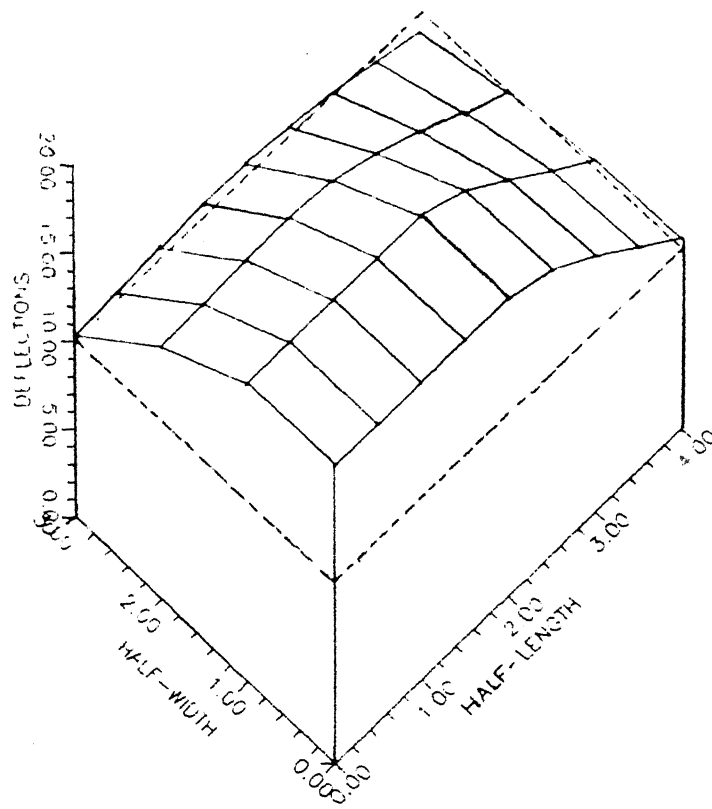


PLATE MODE SHAPE FOR MODEL TYPE I AFTER FIRST ITERATION.

FIG. 3.1

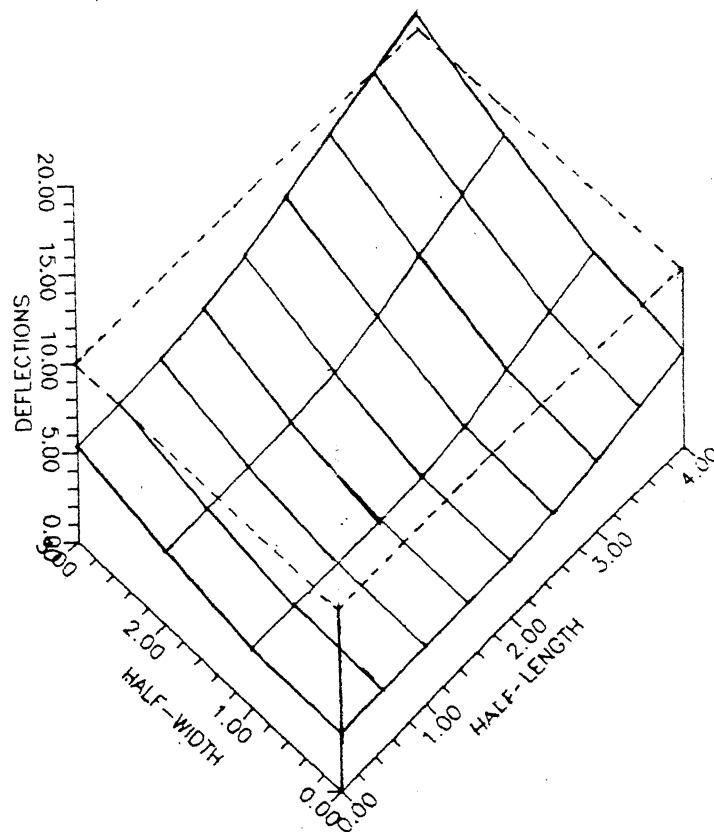


PLATE MODE SHAPE FOR MODEL TYPE I AFTER SECOND ITERATION.

FIG. 3.2

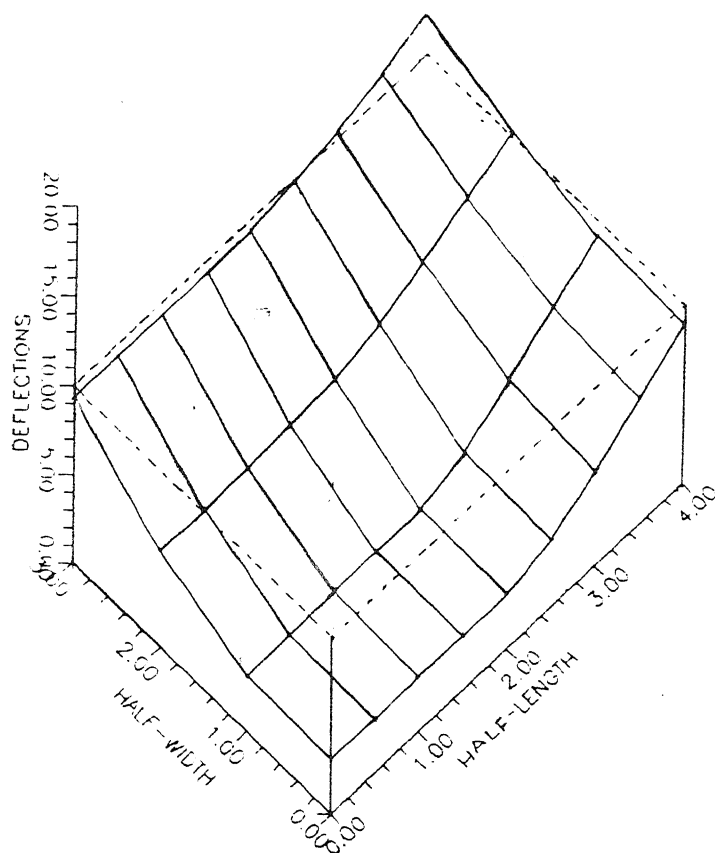


PLATE MODE SHAPE FOR MODEL TYPE I AFTER THIRD ITERATION.

FIG. 3.3.

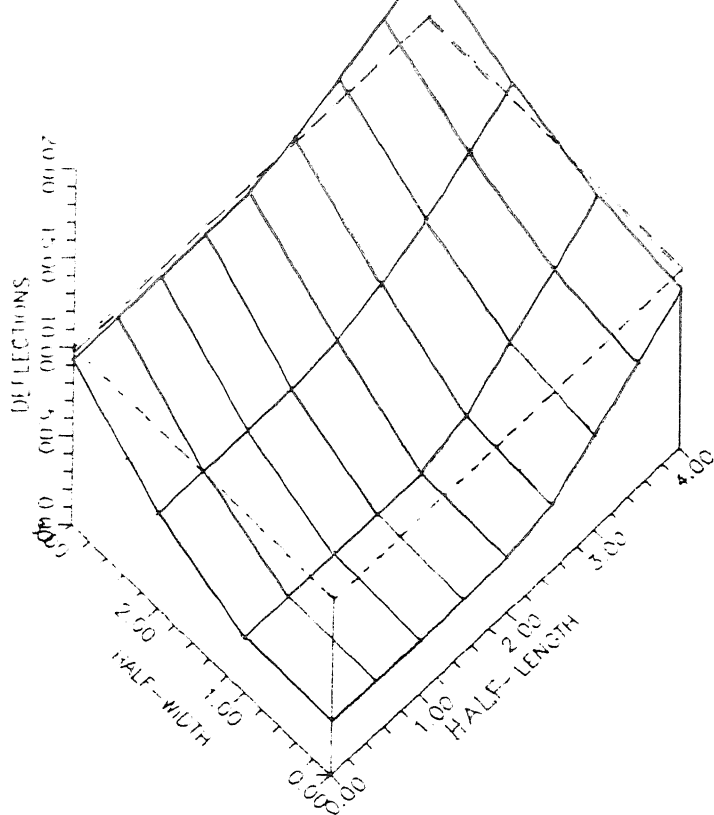
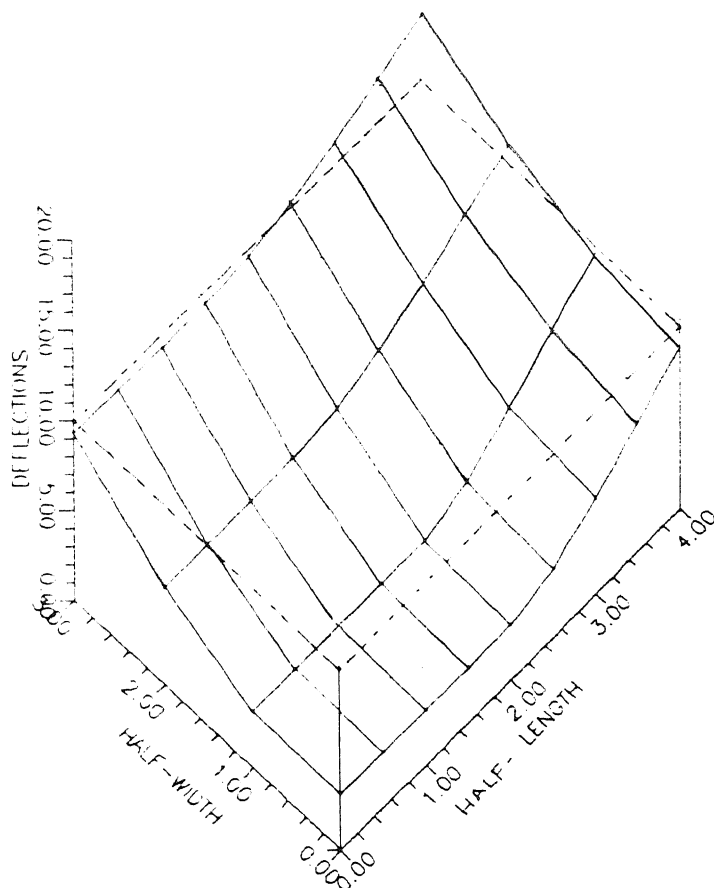


PLATE MODE SHAPE FOR MODEL TYPE I AFTER FOURTH ITERATION.

FIG. 3.4



Pattern 1

FINAL MODE SHAPE OF THE PLATE FOR MODEL TYPE I.

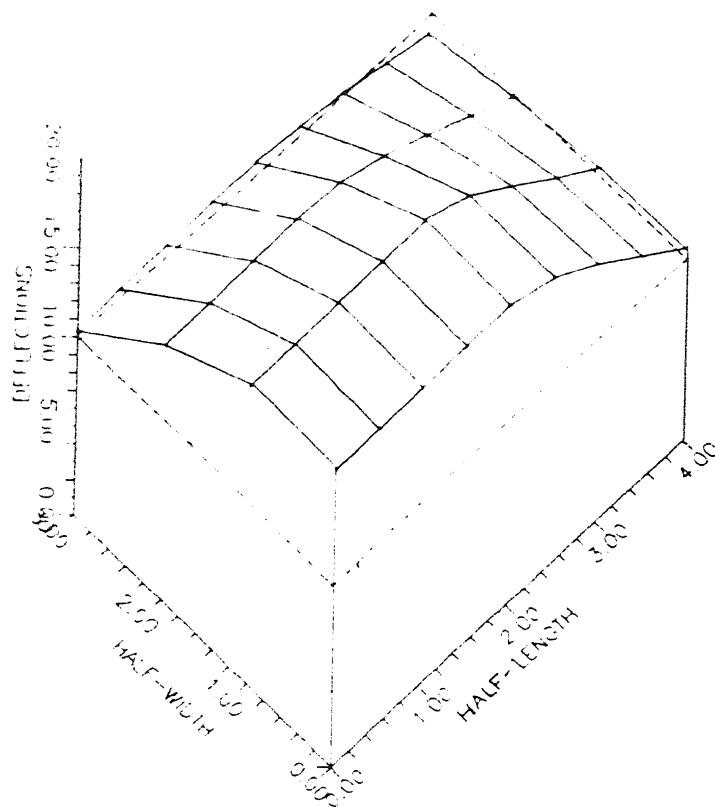


PLATE MODE SHAPE FOR MODEL TYPE II AFTER FIRST ITERATION.

FIG. 3.6

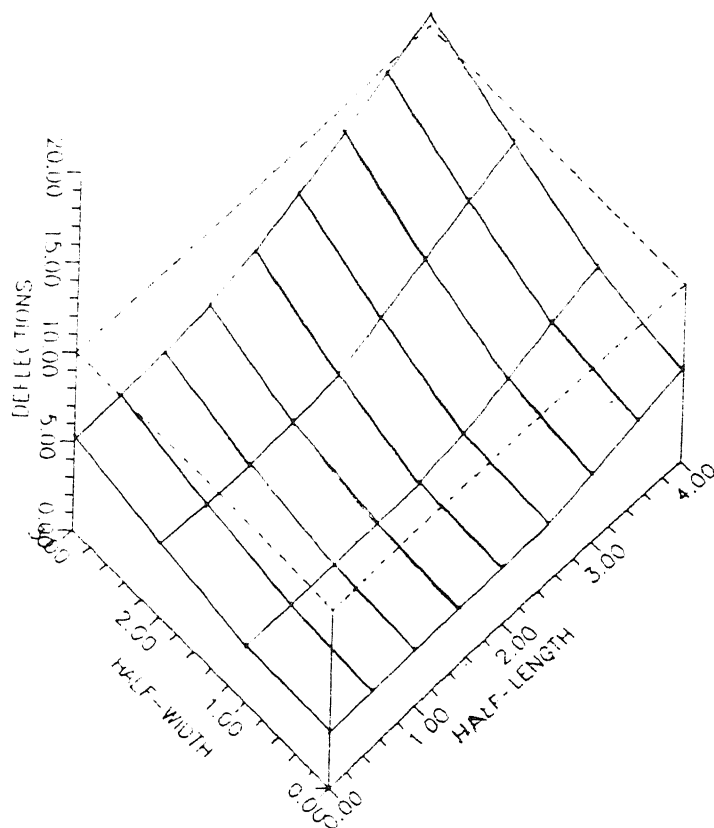


PLATE MODE SHAPE FOR MODEL TYPE II AFTER SECOND ITERATION.

FIG. 3.7

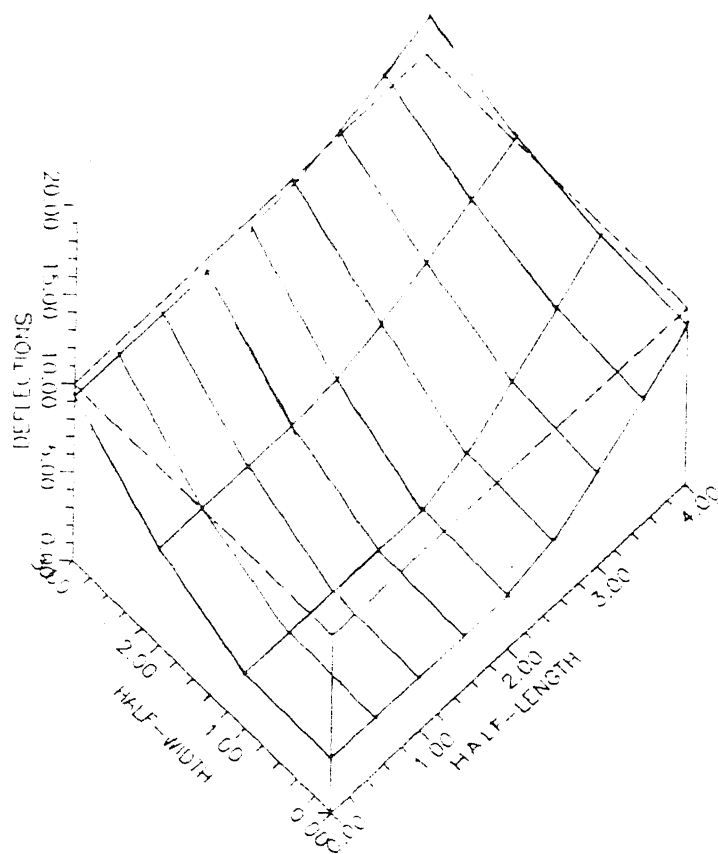


PLATE MODE SHAPE FOR MODEL TYPE II AFTER THIRD ITERATION.

FIG. 38

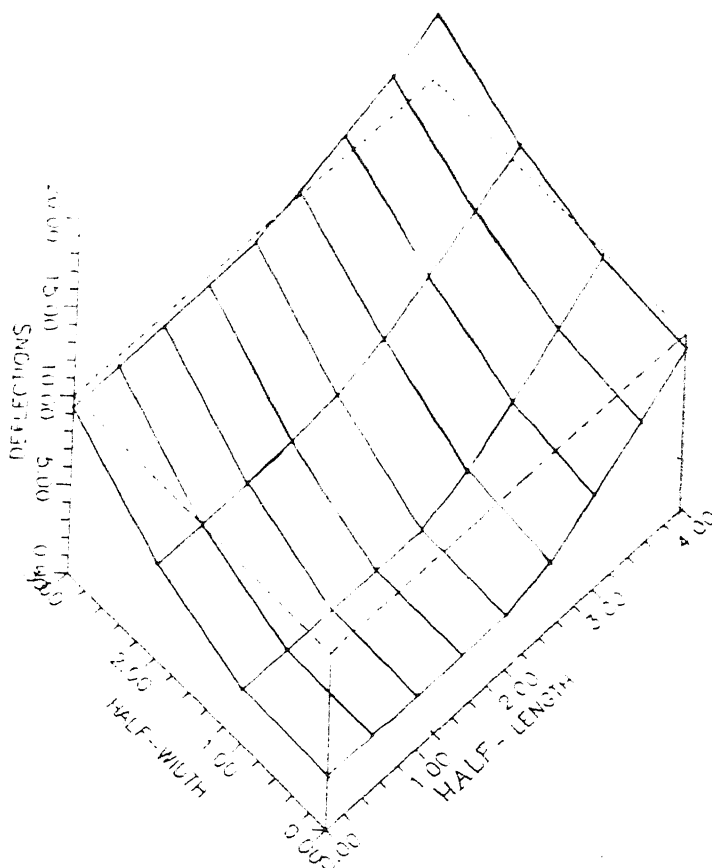
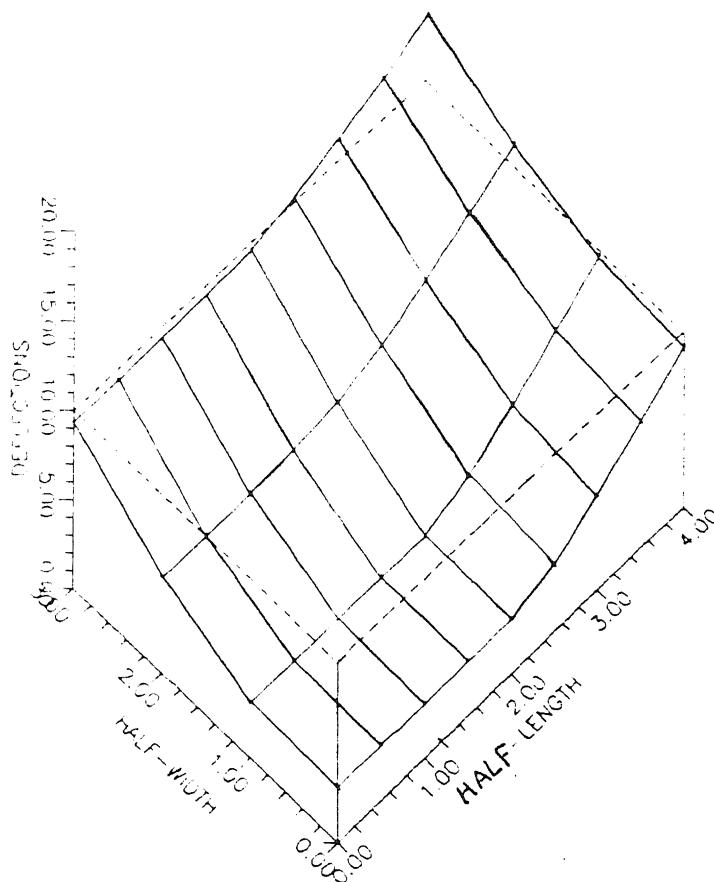


PLATE MODE SHAPE FOR MODEL TYPE II AFTER FOURTH ITERATION.

FIG. 3.9



Pattern I

FINAL MODE SHAPE OF THE PLATE FOR MODEL TYPE II.

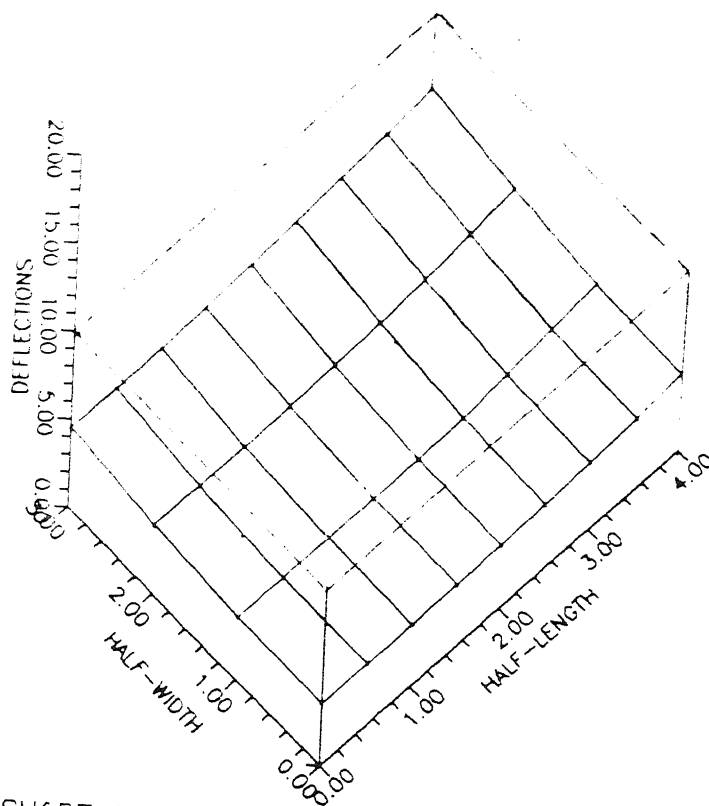


PLATE MODE SHAPE SHOWING SECOND PATTERN (Complete compression) FOR MODEL TYPE I plate under-

FIG. 3.11

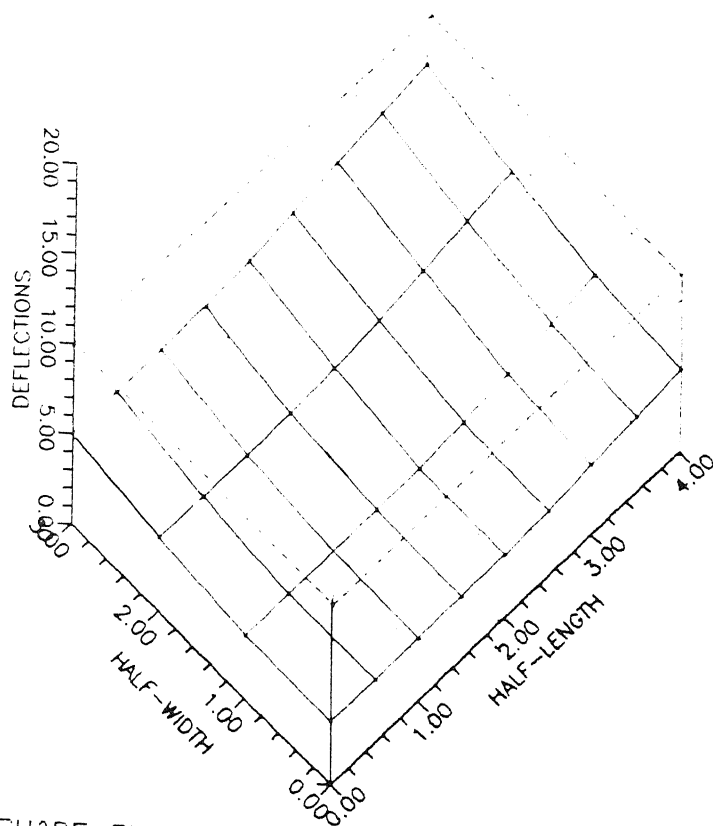


PLATE MODE SHAPE SHOWING SECOND PATTERN (Complete compression) FOR MODEL TYPE II plate under-

FIG. 3.12

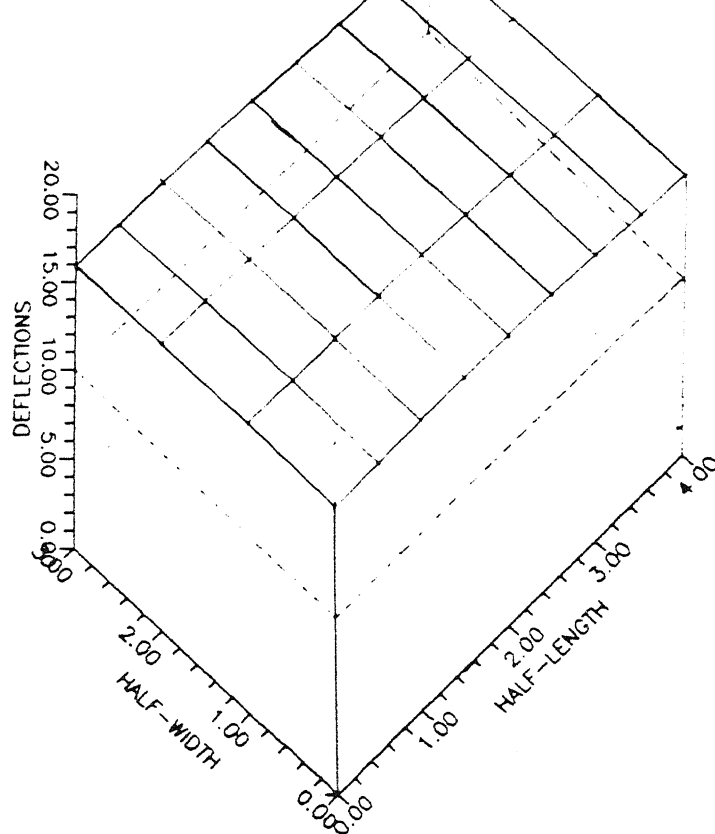


PLATE MODE SHAPE SHOWING THIRD PATTERN (Complete plate under lift-off) FOR MODEL TYPE I **FIG. 3.13**

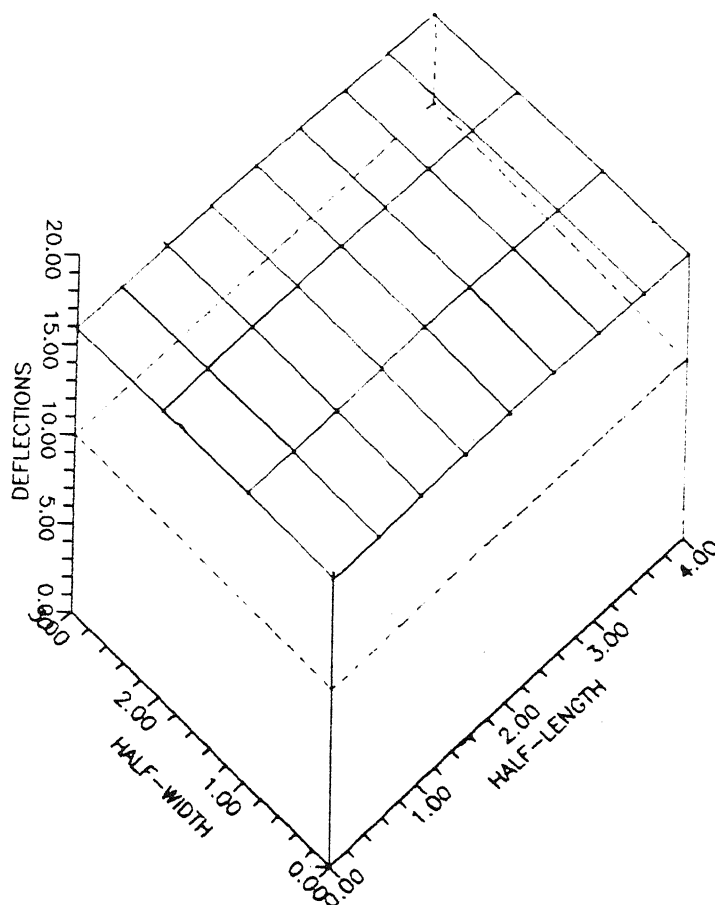
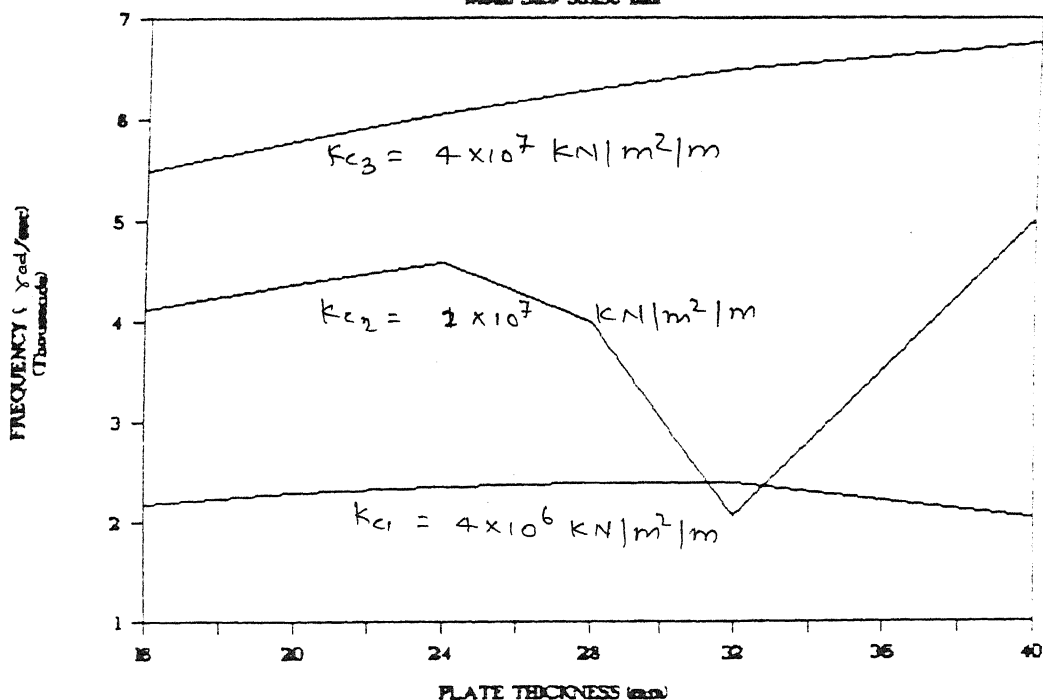


PLATE MODE SHAPE SHOWING THIRD PATTERN (Complete plate under lift-off) FOR MODEL TYPE II

MODEL TYPE I

Mesh Size 50x50 mm

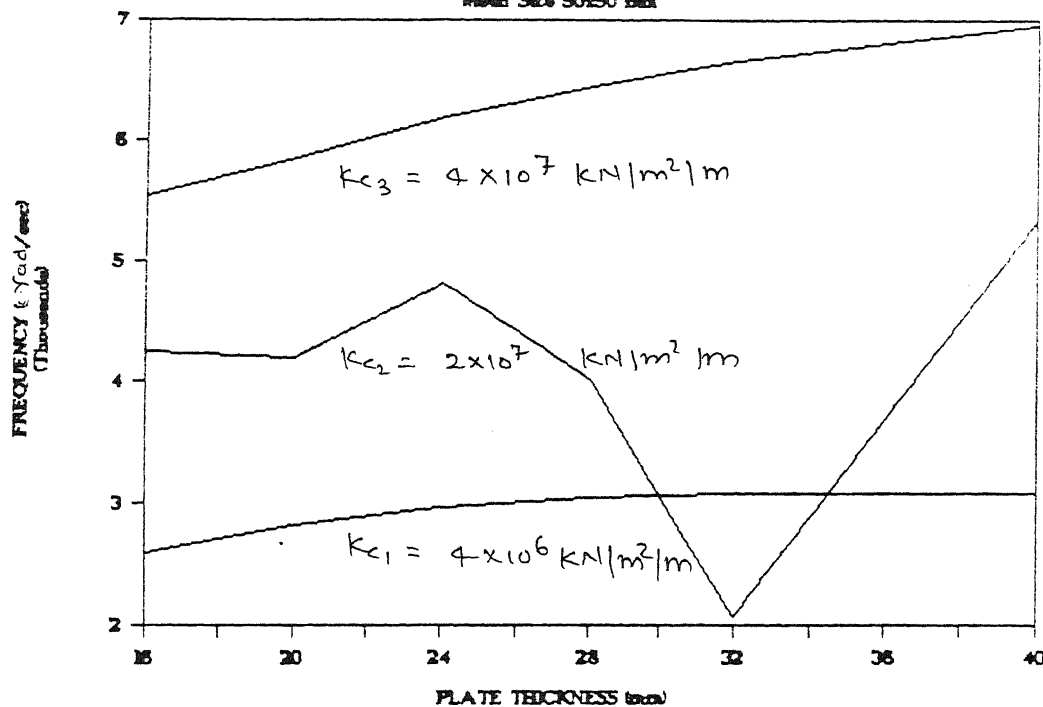


FREQUENCY VS THICKNESS OF PLATE .

FIG. 3.15

MODEL TYPE II

Mesh Size 50x50 mm

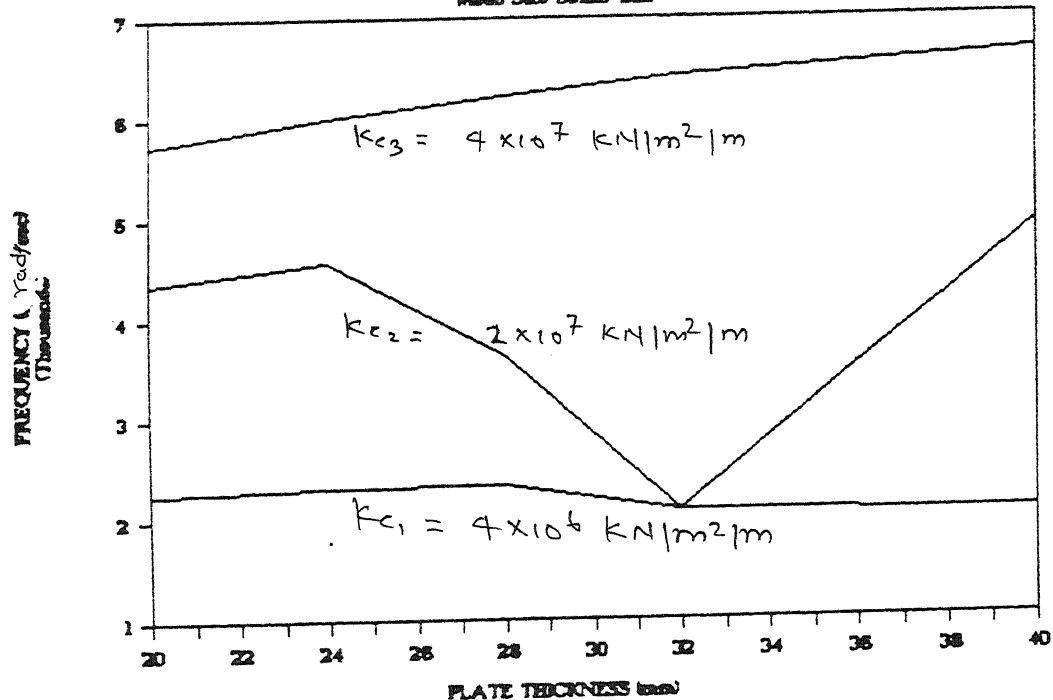


FREQUENCY VS THICKNESS OF PLATE .

FIG. 3.16

MODEL TYPE I

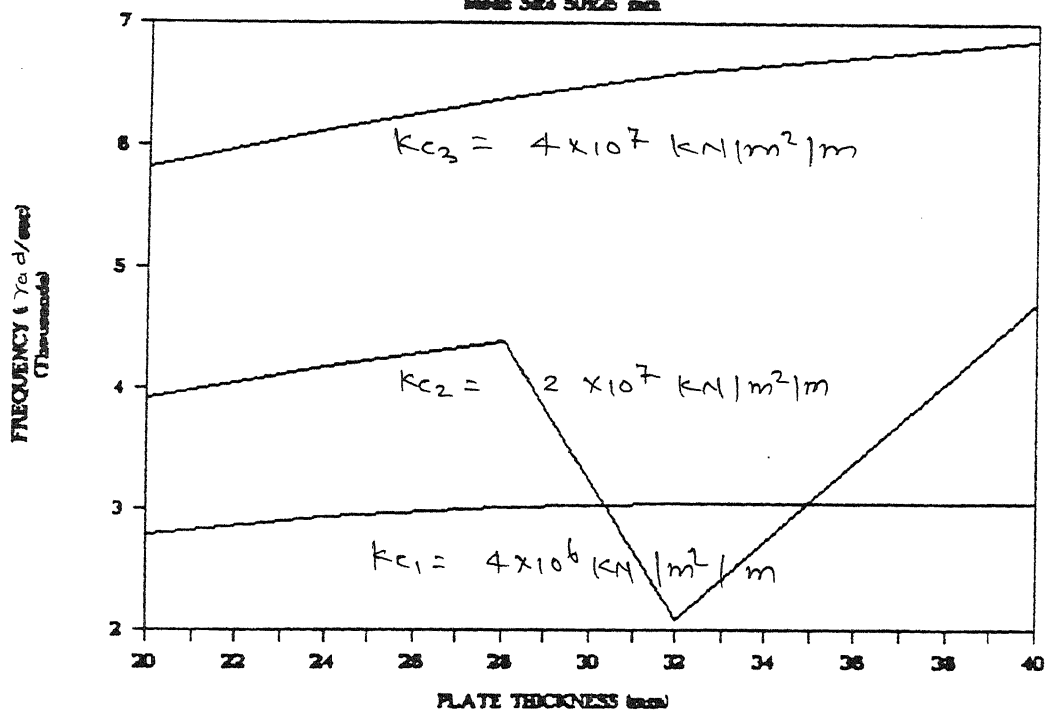
Mesh Size 50x25 mm



FREQUENCY VS THICKNESS OF PLATE
FIG. 3.17

MODEL TYPE II

Mesh Size 50x25 mm



FREQUENCY VS THICKNESS OF PLATE
FIG. 3.18

TABLE 3.1.1:- EIGENVECTOR (MODE SHAPE) FOR MODELTYPE I
FIRST ITERATION

Thickness of plate = 20 mm
 Anchor diameter = 32 mm
 Concrete spring constant = 4.0×10^7 kn/m²
 Anchor spring constant = 8.0×10^4 kn/m

Eigenvalue/frequency :-

Fundamental frequency = 5861 rad/sec
 = 932.8 cycles/sec
 Period = 0.1072 E-02 sec

node displacements / rotations

node number	x- translation	y- translation	z- translation	x- rotation	y- rotation	z- rotation
36	.46614E-19	- .28863E-17	- .14091E+01	- .23883E+02	.26832E+02	.00000E+00
35	.11004E-17	- .26680E-17	- .15577E+00	- .26338E+02	.46911E+02	.00000E+00
34	.10594E-17	- .19735E-17	.66280E+00	- .82040E+01	.67377E+02	.00000E+00
33	.20100E-17	.00000E+00	.86408E+00	.00000E+00	.73704E+02	.00000E+00
32	.75900E-19	- .34689E-17	- .75803E+00	- .33837E+02	.24430E+02	.00000E+00
31	.11140E-17	- .34606E-17	.10315E+01	- .37465E+02	.48343E+02	.00000E+00
30	.16080E-17	- .23773E-17	.23700E+01	- .13969E+02	.71622E+02	.00000E+00
29	.16161E-17	.00000E+00	.27075E+01	.00000E+00	.75468E+02	.00000E+00
28	.21200E-16	- .40324E-17	- .19704E+00	- .46763E+02	.20191E+02	.00000E+00
27	.11059E-17	- .42200E-17	.22473E+01	- .52346E+02	.49277E+02	.00000E+00
26	.12657E-17	- .28799E-17	.41910E+01	- .18234E+02	.76161E+02	.00000E+00
25	.10498E-17	.00000E+00	.45668E+01	.00000E+00	.72524E+02	.00000E+00
24	.38787E-18	- .44213E-17	.25398E+00	- .60436E+02	.15120E+02	.00000E+00
23	.11400E-17	- .45029E-17	.33970E+01	- .68235E+02	.41327E+02	.00000E+00
22	.85280E-18	- .28020E-17	.59647E+01	- .17110E+02	.65714E+02	.00000E+00
21	.35907E-18	.00000E+00	.61695E+01	.00000E+00	.50545E+02	.00000E+00
20	.54562E-18	- .45626E-17	.53938E+00	- .69707E+02	.69910E+01	.00000E+00
19	.10010E-17	- .45725E-17	.41905E+01	- .74945E+02	.23463E+02	.00000E+00
18	.35955E-18	- .60955E-18	.69012E+01	- .23429E-01	.83748E-01	.00000E+00
17	.11697E-18	.00000E+00	.69018E+01	.00000E+00	.67757E-01	.00000E+00
16	.60636E-18	- .44599E-17	.59270E+00	- .76440E+02	- .11483E+01	.00000E+00
15	.63923E-18	- .42548E-17	.44856E+01	- .77618E+02	.43083E+01	.00000E+00
14	.23902E-18	- .34499E-18	.69031E+01	- .13183E-01	.69783E-01	.00000E+00
13	.76983E-19	.00000E+00	.69035E+01	.00000E+00	.61797E-01	.00000E+00
12	.48453E-18	- .42360E-17	.49263E+00	- .79511E+02	- .55340E+01	.00000E+00
11	.31910E-18	- .40444E-17	.44807E+01	- .78864E+02	- .32841E+01	.00000E+00
10	.14017E-18	- .28232E-18	.69047E+01	- .72534E-02	.49843E-01	.00000E+00
9	.63121E-19	.00000E+00	.69049E+01	.00000E+00	.45584E-01	.00000E+00
8	.26684E-18	- .40099E-17	.35135E+00	- .80943E+02	- .47626E+01	.00000E+00
7	.16747E-18	- .38918E-17	.43726E+01	- .80163E+02	- .50124E+01	.00000E+00
6	.64418E-19	- .27122E-18	.69056E+01	- .44804E-02	.25797E-01	.00000E+00
5	.31342E-19	.00000E+00	.69058E+01	.00000E+00	.23865E-01	.00000E+00
4	.00000E+00	- .40112E-17	.28185E+00	- .81683E+02	.00000E+00	.00000E+00
3	.00000E+00	- .36153E-17	.42986E+01	- .80813E+02	.00000E+00	.00000E+00
2	.00000E+00	- .27018E-18	.69059E+01	- .37651E-02	.00000E+00	.00000E+00
1	.00000E+00	.00000E+00	.69061E+01	.00000E+00	.00000E+00	.00000E+00

TABLE 3.1.2:- EIGENVECTOR (MODE SHAPE) FOR MODELTYPE 1
SECOND ITERATION

Thickness of plate = 20 mm
 Anchor diameter = 32 mm
 Concrete spring constant = 4.0×10^7 kn/m²
 Anchor spring constant = 8.0×10^4 kn/m

Eigenvalue/frequency -

Fundamental frequency = 2088 rad/sec
 = 332.4 cycles/sec
 Period = 0.3009 E-02 sec

node displacements / rotations

node number	x- translation	y- translation	z- translation	x- rotation	y- rotation	z- rotation
36	-11150E-17	15447E-17	.66005E+00	.44462E+02	-.39335E+02	.00000E+00
35	-53757E-18	.19286E-17	-.16128E+01	.48136E+02	-.40718E+02	.00000E+00
34	.41616E-19	.66901E-18	-.37369E+01	.30693E+02	-.33427E+02	.00000E+00
33	.51117E-19	.00000E+00	-.45629E+01	.00000E+00	-.28400E+02	.00000E+00
32	-.10804E-17	.11321E-17	-.33203E+00	.45197E+02	-.40767E+02	.00000E+00
31	-.27517E-18	.95592E-18	-.26248E+01	.47214E+02	-.40859E+02	.00000E+00
30	.12936E-18	.53141E-18	-.46145E+01	.27790E+02	-.36405E+02	.00000E+00
29	.12564E-18	.00000E+00	-.53208E+01	.00000E+00	-.30366E+02	.00000E+00
28	-.90231E-18	.63580E-18	-.13935E+01	.45474E+02	-.44796E+02	.00000E+00
27	-.23669E-18	.49384E-18	-.36527E+01	.46360E+02	-.42298E+02	.00000E+00
26	.11462E-18	.29020E-18	-.55271E+01	.22847E+02	-.36223E+02	.00000E+00
25	.15665E-18	.00000E+00	-.60521E+01	.00000E+00	-.25338E+02	.00000E+00
24	-.69266E-18	.32270E-18	-.25309E+01	.41865E+02	-.44038E+02	.00000E+00
23	-.20509E-18	.21761E-18	-.46763E+01	.43957E+02	-.38112E+02	.00000E+00
22	.70428E-19	.11781E-18	-.63497E+01	.14480E+02	-.28307E+02	.00000E+00
21	.11388E-18	.00000E+00	-.65747E+01	.00000E+00	-.12467E+02	.00000E+00
20	-.50208E-18	.14365E-18	-.35212E+01	.36594E+02	-.33820E+02	.00000E+00
19	-.16392E-18	.68845E-19	-.54556E+01	.37569E+02	-.24340E+02	.00000E+00
18	.15140E-19	.47919E-20	-.67592E+01	.11001E-01	-.24519E-01	.00000E+00
17	.19628E-19	.00000E+00	-.67595E+01	.00000E+00	-.16695E-01	.00000E+00
16	-.33973E-18	.50489E-19	-.41870E+01	.32136E+02	-.19940E+02	.00000E+00
15	-.12589E-18	-.19504E-21	-.58596E+01	.32030E+02	-.92749E+01	.00000E+00
14	.11347E-19	-.17347E-20	-.67598E+01	.55912E-02	-.19788E-01	.00000E+00
13	.15569E-19	.00000E+00	-.67599E+01	.00000E+00	-.16005E-01	.00000E+00
12	-.20760E-18	.88502E-20	-.45294E+01	.28056E+02	-.81690E+01	.00000E+00
11	-.85597E-19	-.27112E-19	-.59802E+01	.28072E+02	-.42617E+00	.00000E+00
10	.75994E-20	-.43130E-20	-.67602E+01	.28469E-02	-.13709E-01	.00000E+00
9	.10406E-19	.00000E+00	-.67603E+01	.00000E+00	-.11936E-01	.00000E+00
8	-.98024E-19	-.68057E-20	-.46400E+01	.25511E+02	-.16198E+01	.00000E+00
7	-.43683E-19	-.36205E-19	-.59386E+01	.26444E+02	.39763E+01	.00000E+00
6	.38057E-20	-.52015E-20	-.67605E+01	.19072E-02	-.68864E-02	.00000E+00
5	.51575E-20	.00000E+00	-.67605E+01	.00000E+00	-.62836E-02	.00000E+00
4	.00000E+00	-.10628E-19	-.46476E+01	.25104E+02	.00000E+00	.00000E+00
3	.00000E+00	-.35038E-19	-.58756E+01	.26184E+02	.00000E+00	.00000E+00
2	.00000E+00	-.54056E-20	-.67605E+01	.18029E-02	.00000E+00	.00000E+00
1	.00000E+00	.00000E+00	-.67606E+01	.00000E+00	.00000E+00	.00000E+00

TABLE 3.1.3:- EIGENVECTOR (MODE SHAPE) FOR MODEL TYPE I
THIRD ITERATION

Thickness of plate = 20 mm
Anchor diameter = 32 mm
Concrete spring constant = $4.0 \text{ E}+07 \text{ kn/m}^2$
Anchor spring constant = $8.0 \text{ E}+04 \text{ kn/m}$

Eigenvalue/frequency -

Fundamental frequency = 5744 rad/sec
= 914.2 cycles/sec
Period = 0.1094 E-02 sec

node displacements / rotations

node number	x- translation	y- translation	z- translation	x- rotation	y- rotation	z- rotation
36	19010E-19	28822E-17	20543E+01	36681E+02	-41011E+02	00000E+00
35	-11757E-17	26329E-17	26578E+00	33274E+02	-52183E+02	00000E+00
34	-19134E-17	20050E-17	-71584E+00	89623E+01	-67902E+02	00000E+00
33	-20602E-17	00000E+00	-91903E+00	00000E+00	-73327E+02	00000E+00
32	-12617E-19	35094E-17	10595E+01	41777E+02	-36636E+02	00000E+00
31	-11330E-17	35108E-17	-10626E+01	41130E+02	-53599E+02	00000E+00
30	-16608E-17	24179E-17	-24411E+01	13320E+02	-71832E+02	00000E+00
29	-16577E-17	00000E+00	-27548E+01	00000E+00	-74852E+02	00000E+00
28	-16715E-18	41184E-17	25038E+00	51030E+02	-27327E+02	00000E+00
27	-11250E-17	43285E-17	-23760E+01	53297E+02	-50909E+02	00000E+00
26	-13056E-17	29366E-17	-42567E+01	16766E+02	-74968E+02	00000E+00
25	-10789E-17	00000E+00	-45962E+01	00000E+00	-71558E+02	00000E+00
24	-36268E-18	45304E-17	-31963E+00	62529E+02	-17978E+02	00000E+00
23	-11617E-17	47246E-17	-35210E+01	67340E+02	-39624E+02	00000E+00
22	-87644E-18	28541E-17	-59896E+01	15854E+02	-63618E+02	00000E+00
21	-37174E-18	00000E+00	-61746E+01	00000E+00	-49862E+02	00000E+00
20	-53145E-18	46781E-17	-65131E+00	70436E+02	-83060E+01	00000E+00
19	-10182E-17	46938E-17	-42794E+01	73079E+02	-22260E+02	00000E+00
18	-36827E-18	61988E-18	-68941E+01	21646E-01	-82542E-01	00000E+00
17	-12113E-18	00000E+00	-68947E+01	00000E+00	-67279E-01	00000E+00
16	-59295E-18	45806E-17	-74231E+00	76434E+02	-75741E+00	00000E+00
15	-65183E-18	43622E-17	-45704E+01	74978E+02	-50184E+01	00000E+00
14	-24529E-18	35098E-18	-68960E+01	11838E-01	-69013E-01	00000E+00
13	-80286E-19	00000E+00	-68963E+01	00000E+00	-61236E-01	00000E+00
12	-46824E-18	43707E-17	-70172E+00	79326E+02	26250E+01	00000E+00
11	-37919E-18	41709E-17	-46072E+01	75401E+02	58445E+00	00000E+00
10	-14433E-18	28797E-18	-68975E+01	58993E-02	-49595E-01	00000E+00
9	-65772E-19	00000E+00	-68977E+01	00000E+00	-45203E-01	00000E+00
8	-25432E-18	42236E-17	-63116E+00	80906E+02	22683E+01	00000E+00
7	-17362E-18	40502E-17	-45801E+01	75889E+02	11129E+01	00000E+00
6	-66563E-19	27751E-18	-68985E+01	28199E-02	-25836E-01	00000E+00
5	-37837E-19	00000E+00	-68986E+01	00000E+00	-23697E-01	00000E+00
4	00000E+00	41737E-17	-60004E+00	81427E+02	00000E+00	00000E+00
3	00000E+00	39921E-17	-45648E+01	76124E+02	00000E+00	00000E+00
2	00000E+00	27682E-18	-68988E+01	18835E-02	00000E+00	00000E+00
1	00000E+00	00000E+00	-68989E+01	00000E+00	00000E+00	00000E+00

TABLE 3.1.4:- EIGENVECTOR (MODE SHAPE) FOR MODEL TYPE I
FOURTH ITERATION

Thickness of plate = 20 mm
 Anchor diameter = 32 mm
 Concrete spring constant = $4.0 \text{ E}+07 \text{ kn/m}^2$
 Anchor spring constant = $8.0 \text{ E}+04 \text{ kn/m}$

Eigenvalue/frequency :-

Fundamental frequency = 5731 rad/sec
 = 912.2 cycles/sec
 Period = $0.1096 \text{ E}-02 \text{ sec}$

node displacements / rotations							
node number	x-translation	y-translation	z-translation	x-rotation	y-rotation	z-rotation	
36	70170E-19	25403E-17	.35298E+01	.52733E+02	-.51597E+02	.00000E+00	
35	-.10585E-17	24124E-17	.99084E+00	.46484E+02	-.59442E+02	.00000E+00	
34	-.17807E-17	17477E-17	-.51244E+00	.15542E+02	-.69883E+02	.00000E+00	
33	-.19803E-17	.00000E+00	-.87630E+00	.00000E+00	-.73658E+02	.00000E+00	
32	.59194E-19	.31540E-17	.22523E+01	.56238E+02	-.49249E+02	.00000E+00	
31	-.10264E-17	30745E-17	-.52172E+00	.52229E+02	-.61050E+02	.00000E+00	
30	-.15623E-17	22042E-17	-.23007E+01	.18336E+02	-.74367E+02	.00000E+00	
29	-.16147E-17	.00000E+00	-.27329E+01	.00000E+00	-.75596E+02	.00000E+00	
28	-.37443E-19	37592E-17	.10944E+01	.61481E+02	-.41790E+02	.00000E+00	
27	-.10378E-17	.39334E-17	-.20180E+01	.61534E+02	-.57689E+02	.00000E+00	
26	-.12564E-17	27478E-17	-.41820E+01	.20036E+02	-.77228E+02	.00000E+00	
25	-.10639E-17	.00000E+00	-.45945E+01	.00000E+00	-.71878E+02	.00000E+00	
24	-.20241E-18	42433E-17	.17804E+00	.68443E+02	-.30274E+02	.00000E+00	
23	-.10930E-17	44415E-17	-.33212E+01	.72621E+02	-.44944E+02	.00000E+00	
22	-.86006E-18	27288E-17	-.59666E+01	.17439E+02	-.65101E+02	.00000E+00	
21	-.38021E-18	.00000E+00	-.61785E+01	.00000E+00	-.49521E+02	.00000E+00	
20	-.37351E-18	44867E-17	-.40892E+00	.73241E+02	-.16282E+02	.00000E+00	
19	-.97122E-18	.45189E-17	-.41914E+01	.75905E+02	-.25583E+02	.00000E+00	
18	-.36654E-18	.60536E-18	-.68951E+01	.22240E-01	-.83266E-01	.00000E+00	
17	-.12645E-18	.00000E+00	-.68957E+01	.00000E+00	-.67488E-01	.00000E+00	
16	-.46288E-18	.44745E-17	-.65137E+00	.77501E+02	-.50849E+01	.00000E+00	
15	-.62495E-18	.42877E-17	-.45440E+01	.76248E+02	-.65751E+01	.00000E+00	
14	-.24549E-18	.34644E-18	-.68971E+01	.12016E-01	-.69567E-01	.00000E+00	
13	-.85430E-19	.00000E+00	-.68974E+01	.00000E+00	-.61529E-01	.00000E+00	
12	-.37994E-18	.43239E-17	-.68735E+00	.79482E+02	-.62708E+00	.00000E+00	
11	-.36372E-18	.41470E-17	-.46082E+01	.75737E+02	-.24810E-01	.00000E+00	
10	-.14525E-18	.28723E-18	-.68986E+01	.58723E-02	-.49967E-01	.00000E+00	
9	-.69287E-19	.00000E+00	-.68988E+01	.00000E+00	-.45439E-01	.00000E+00	
8	-.21042E-18	.42082E-17	-.64913E+00	.80681E+02	.15431E+01	.00000E+00	
7	-.16627E-18	.40515E-17	-.45911E+01	.75787E+02	.92680E+00	.00000E+00	
6	-.67246E-19	.27823E-18	-.68996E+01	.27068E-02	-.26020E-01	.00000E+00	
5	-.39577E-19	.00000E+00	-.68997E+01	.00000E+00	-.23825E-01	.00000E+00	
4	.00000E+00	.41679E-17	-.62650E+00	.81103E+02	.00000E+00	.00000E+00	
3	.00000E+00	.40007E-17	-.45782E+01	.75900E+02	.00000E+00	.00000E+00	
2	.00000E+00	.27793E-18	-.68999E+01	.17477E-02	.00000E+00	.00000E+00	
1	.00000E+00	.00000E+00	-.69000E+01	.00000E+00	.00000E+00	.00000E+00	

TABLE 3.1.5:- EIGENVECTOR (MODE SHAPE) FOR MODEL TYPE I
FIFTH ITERATION

Thickness of plate = 20 mm
 Anchor diameter = 32 mm
 Concrete spring constant = $4.0 \text{ E}+07 \text{ kn/m}^2/\text{m}$
 Anchor spring constant = $8.0 \text{ E}+04 \text{ kn/m}$

Eigenvalue/frequency :-

Fundamental frequency = 5731 rad/sec
 = 912.2 cycles/sec
 Period = $0.1096 \text{ E}-02 \text{ sec}$

node displacements / rotations						
node number	x- translation	y- translation	z- translation	x- rotation	y- rotation	z- rotation
36	.82500E-19	25012E-17	.36209E+01	.53985E+02	-.51625E+02	.00000E+00
35	-.10463E-17	.23774E-17	.10259E+01	.47413E+02	-.59469E+02	.00000E+00
34	-.17695E-17	.17263E-17	-.50972E+00	.15895E+02	-.69822E+02	.00000E+00
33	-.19724E-17	.00000E+00	-.88194E+00	.00000E+00	-.73566E+02	.00000E+00
32	.72559E-19	.31161E-17	.23421E+01	.57494E+02	-.49333E+02	.00000E+00
31	-.10136E-17	.30342E-17	-.48807E+00	.53137E+02	-.61149E+02	.00000E+00
30	-.15545E-17	.21845E-17	-.22973E+01	.18654E+02	-.74368E+02	.00000E+00
29	-.16100E-17	.00000E+00	-.27368E+01	.00000E+00	-.75533E+02	.00000E+00
28	-.21870E-19	.37238E-17	.11806E+01	.62748E+02	-.42027E+02	.00000E+00
27	-.10297E-17	.38964E-17	-.19877E+01	.62408E+02	-.57880E+02	.00000E+00
26	-.12020E-17	.27302E-17	-.41793E+01	.20293E+02	-.77272E+02	.00000E+00
25	-.10623E-17	.00000E+00	-.45970E+01	.00000E+00	-.71809E+02	.00000E+00
24	-.18080E-18	.42139E-17	.25351E+00	.69674E+02	-.30974E+02	.00000E+00
23	-.10805E-17	.44122E-17	-.32968E+01	.73410E+02	-.45225E+02	.00000E+00
22	-.85827E-18	.27164E-17	-.59654E+01	.17600E+02	-.65165E+02	.00000E+00
21	-.38090E-18	.00000E+00	-.61795E+01	.00000E+00	-.49438E+02	.00000E+00
20	-.35763E-18	.44651E-17	-.35661E+00	.74022E+02	-.17260E+02	.00000E+00
19	-.96081E-18	.44991E-17	-.41751E+01	.76464E+02	-.25881E+02	.00000E+00
18	-.36622E-18	.60372E-18	-.68952E+01	.22344E-01	-.83361E-01	.00000E+00
17	-.12689E-18	.00000E+00	-.68958E+01	.00000E+00	-.67525E-01	.00000E+00
16	-.44958E-18	.44606E-17	-.62135E+00	.77938E+02	-.58420E+01	.00000E+00
15	-.62178E-18	.42764E-17	-.45349E+01	.76596E+02	-.68032E+01	.00000E+00
14	-.24541E-18	.34584E-18	-.68972E+01	.12065E-01	-.69655E-01	.00000E+00
13	-.85883E-19	.00000E+00	-.68975E+01	.00000E+00	-.61580E-01	.00000E+00
12	-.37065E-18	.43159E-17	-.67271E+00	.79689E+02	.16456E+00	.00000E+00
11	-.36195E-18	.41414E-17	-.46040E+01	.75922E+02	-.16596E+00	.00000E+00
10	-.14508E-18	.28702E-18	-.68987E+01	.58868E-02	-.50036E-01	.00000E+00
9	-.69597E-19	.00000E+00	-.68989E+01	.00000E+00	-.45482E-01	.00000E+00
8	-.28509E-18	.42038E-17	-.64287E+00	.80767E+02	.13325E+01	.00000E+00
7	-.16547E-18	.40490E-17	-.45896E+01	.75877E+02	.86239E+00	.00000E+00
6	-.67289E-19	.27819E-18	-.68997E+01	.27024E-02	-.26057E-01	.00000E+00
5	-.38731E-19	.00000E+00	-.68998E+01	.00000E+00	-.23849E-01	.00000E+00
4	.00000E+00	.41646E-17	-.62285E+00	.81153E+02	.00000E+00	.00000E+00
3	.00000E+00	.39993E-17	-.45776E+01	.75960E+02	.00000E+00	.00000E+00
2	.00000E+00	.27793E-18	-.69000E+01	.17372E-02	.00000E+00	.00000E+00
1	.00000E+00	.00000E+00	-.69001E+01	.00000E+00	.00000E+00	.00000E+00

TABLE 3.2.1:- EIGENVECTOR (MODE SHAPE) FOR MODEL TYPE II
FIRST ITERATION

Thickness of plate = 20 mm
Anchor diameter = 32 mm
Concrete spring constant = 4.0 E+07 kn/m²
Anchor spring constant = 8.0 E+04 kn/m

Eigenvalue/frequency :-

Fundamental frequency = 2088 rad/sec
= 332.4 cycles/sec
Period = 0.3009 E-02 sec

node displacements / rotations						
node number	x-translation	y-translation	z-translation	x-rotation	y-rotation	z-rotation
36	-11150E-17	15447E-17	.66005E+00	.44462E+02	-.39335E+02	.00000E+00
35	-33757E-18	19288E-17	-.16128E+01	.48136E+02	-.40718E+02	.00000E+00
34	.41616E-19	.66901E-18	-.37369E+01	.30693E+02	-.33427E+02	.00000E+00
33	.51117E-19	.00000E+00	-.45629E+01	.00000E+00	-.28400E+02	.00000E+00
32	-.10808E-17	.11321E-17	-.33203E+00	.45197E+02	-.40767E+02	.00000E+00
31	-.27517E-18	.95592E-18	-.26248E+01	.47214E+02	-.40859E+02	.00000E+00
30	.12938E-18	.53141E-18	-.46145E+01	.27790E+02	-.36405E+02	.00000E+00
29	.12564E-18	.00000E+00	-.53208E+01	.00000E+00	-.30366E+02	.00000E+00
28	-.90231E-18	.63580E-18	-.13935E+01	.45474E+02	-.44796E+02	.00000E+00
27	-.23609E-18	.49354E-18	-.36527E+01	.46360E+02	-.42298E+02	.00000E+00
26	.11462E-18	.29020E-18	-.55271E+01	.22847E+02	-.36223E+02	.00000E+00
25	.15565E-18	.00000E+00	-.60521E+01	.00000E+00	-.25338E+02	.00000E+00
24	-.69266E-18	.32270E-18	-.25309E+01	.41865E+02	-.44038E+02	.00000E+00
23	-.20509E-18	.21761E-18	-.46763E+01	.43957E+02	-.38112E+02	.00000E+00
22	.70428E-19	.11781E-18	-.63497E+01	.14480E+02	-.28307E+02	.00000E+00
21	.11308E-18	.00000E+00	-.65747E+01	.00000E+00	-.12467E+02	.00000E+00
20	-.50208E-18	.14365E-18	-.35212E+01	.36594E+02	-.33820E+02	.00000E+00
19	-.16392E-18	.68845E-19	-.54556E+01	.37569E+02	-.24340E+02	.00000E+00
18	.15140E-19	.47919E-20	-.67592E+01	.11001E-01	-.24519E-01	.00000E+00
17	.19628E-19	.00000E+00	-.67595E+01	.00000E+00	-.16695E-01	.00000E+00
16	-.33973E-18	.50489E-19	-.41870E+01	.32136E+02	-.19940E+02	.00000E+00
15	-.12589E-18	-.19504E-21	-.58596E+01	.32030E+02	-.92749E+01	.00000E+00
14	.11347E-19	-.17347E-20	-.67598E+01	.55912E-02	-.19788E-01	.00000E+00
13	.15565E-19	.00000E+00	-.67599E+01	.00000E+00	-.16005E-01	.00000E+00
12	-.20760E-18	.68502E-20	-.45294E+01	.28056E+02	-.81690E+01	.00000E+00
11	-.85997E-19	-.27112E-19	-.59802E+01	.28072E+02	-.42617E+00	.00000E+00
10	.75994E-20	-.43130E-20	-.67602E+01	.28469E-02	-.13709E-01	.00000E+00
9	.10406E-19	.00000E+00	-.67603E+01	.00000E+00	-.11936E-01	.00000E+00
8	-.98024E-19	-.68057E-20	-.46400E+01	.25511E+02	-.16198E+01	.00000E+00
7	-.43683E-19	-.36205E-19	-.59386E+01	.26444E+02	.39763E+01	.00000E+00
6	.38057E-20	-.52015E-20	-.67605E+01	.19072E-02	-.68864E-02	.00000E+00
5	.51575E-20	.00000E+00	-.67605E+01	.00000E+00	-.62836E-02	.00000E+00
4	.00000E+00	-.10628E-19	-.46476E+01	.25104E+02	.00000E+00	.00000E+00
3	.00000E+00	-.38038E-19	-.58756E+01	.26184E+02	.00000E+00	.00000E+00
2	.00000E+00	-.54056E-20	-.67605E+01	.18029E-02	.00000E+00	.00000E+00
1	.00000E+00	.00000E+00	-.67606E+01	.00000E+00	.00000E+00	.00000E+00

TABLE 3.2.2:- EIGENVECTOR (MODE SHAPE) FOR MODEL TYPE II
SECOND ITERATION

Thickness of plate = 20 mm
 Anchor diameter = 32 mm
 Concrete spring constant = $4.0 \text{ E}+07 \text{ kn/m}^2$
 Anchor spring constant = $8.0 \text{ E}+04 \text{ kn/m}$

Eigenvalue/frequency :-

Fundamental frequency = 5847 rad/sec
 = 930.5 cycles/sec
 Period = $0.1075 \text{ E}-02 \text{ sec}$

node displacements / rotations						
node number	x- translation	y- translation	z- translation	x- rotation	y- rotation	z- rotation
36	31847E-18	22817E-17	23672E+01	36346E+02	-42327E+02	.00000E+00
35	-10117E-17	19985E-17	53315E+00	35557E+02	-52007E+02	.00000E+00
34	-18254E-17	16713E-17	-58554E+00	11735E+02	-68244E+02	.00000E+00
33	-26015E-17	00000E+00	-86447E+00	.00000E+00	-73577E+02	.00000E+00
32	29010E-18	29908E-17	13417E+01	40928E+02	-37894E+02	.00000E+00
31	-99048E-18	29991E-17	-79240E+00	43269E+02	-53423E+02	.00000E+00
30	-16150E-17	21372E-17	-23243E+01	16133E+02	-72723E+02	.00000E+00
29	-16435E-17	.00000E+00	-27120E+01	.00000E+00	-75528E+02	.00000E+00
28	11964E-18	37059E-17	49654E+00	50156E+02	-29203E+02	.00000E+00
27	-10021E-17	39295E-17	-21250E+01	55523E+02	-53020E+02	.00000E+00
26	-12867E-17	27257E-17	-41730E+01	19337E+02	-76878E+02	.00000E+00
25	-10955E-17	.00000E+00	-45742E+01	.00000E+00	-72393E+02	.00000E+00
24	-11120E-18	42278E-17	-12923E+00	61759E+02	-20187E+02	.00000E+00
23	-10554E-17	44473E-17	-33494E+01	69706E+02	-43421E+02	.00000E+00
22	-87057E-18	27253E-17	-59610E+01	17495E+02	-65933E+02	.00000E+00
21	-39840E-18	.00000E+00	-61734E+01	.00000E+00	-50239E+02	.00000E+00
20	-33194E-18	44617E-17	-50538E+00	70015E+02	-93708E+01	.00000E+00
19	-93422E-18	45067E-17	-41806E+01	75433E+02	-24372E+02	.00000E+00
18	-36421E-18	60392E-18	-69011E+01	23437E-01	-83236E-01	.00000E+00
17	-12509E-18	.00000E+00	-69017E+01	.00000E+00	-67225E-01	.00000E+00
16	-45538E-18	44174E-17	-59719E+00	76348E+02	27781E+00	.00000E+00
15	-59048E-18	42396E-17	-44900E+01	77647E+02	-45652E+01	.00000E+00
14	-24294E-18	34444E-18	-65030E+01	13167E-01	-69309E-01	.00000E+00
13	-83707E-19	.00000E+00	-65034E+01	.00000E+00	-61332E-01	.00000E+00
12	-39022E-18	42247E-17	-50906E+00	79297E+02	53416E+01	.00000E+00
11	-33644E-18	40520E-17	-44882E+01	78708E+02	32765E+01	.00000E+00
10	-14301E-18	28373E-18	-69045E+01	72566E-02	-49474E-01	.00000E+00
9	-67642E-19	.00000E+00	-69048E+01	.00000E+00	-45237E-01	.00000E+00
8	-22189E-18	40723E-17	-36939E+00	80719E+02	47708E+01	.00000E+00
7	-15090E-18	39083E-17	-43796E+01	79958E+02	50512E+01	.00000E+00
6	-65099E-19	27333E-18	-69055E+01	45083E-02	-25594E-01	.00000E+00
5	-38582E-19	.00000E+00	-69056E+01	.00000E+00	-23679E-01	.00000E+00
4	.00000E+00	40174E-17	-29956E+00	81464E+02	.00000E+00	.00000E+00
3	.00000E+00	38341E-17	-43051E+01	80603E+02	.00000E+00	.00000E+00
2	.00000E+00	27247E-18	-69058E+01	38034E-02	.00000E+00	.00000E+00
1	.00000E+00	.00000E+00	-69059E+01	.00000E+00	.00000E+00	.00000E+00

TABLE 3.2.3:- EIGENVECTOR (MODE SHAPE) FOR MODELTYPE II
THIRD ITERATION

Thickness of plate = 20 mm
 Anchor diameter = 32 mm
 Concrete spring constant = 4.0×10^7 kn/m²
 Anchor spring constant = 8.0×10^4 kn/m

Eigenvalue/frequency -

Fundamental frequency = 5823 rad/sec
 = 926.8 cycles/sec
 Period = 0.1079×10^{-2} sec

node displacements / rotations						
node number	x- translation	y- translation	z- translation	x- rotation	y- rotation	z- rotation
31	140820E-18	20448E-17	42956E+01	57509E+02	-56588E+02	.00000E+00
32	-907400E-18	19605E-17	14810E+01	52804E+02	-62360E+02	.00000E+00
34	-160040E-17	14093E-17	-32111E+00	20228E+02	-71112E+02	.00000E+00
37	-109790E-17	00000E+00	-80699E+00	.00000E+00	-74125E+02	.00000E+00
38	246970E-18	26873E-17	28911E+01	60104E+02	-54626E+02	.00000E+00
39	-890000E-18	25701E-17	-10457E+00	57717E+02	-63890E+02	.00000E+00
30	-147790E-17	19359E-17	-21486E+01	22441E+02	-76205E+02	.00000E+00
09	-157470E-17	00000E+00	-26844E+01	.00000E+00	-76547E+02	.00000E+00
01	171130E-18	33422E-17	15875E+01	64276E+02	-48143E+02	.00000E+00
17	-901900E-18	35010E-17	-16861E+01	66200E+02	-61919E+02	.00000E+00
16	-121270E-17	25276E-17	-40844E+01	23287E+02	-79709E+02	.00000E+00
20	-100990E-17	00000E+00	-45731E+01	.00000E+00	-72755E+02	.00000E+00
24	362080E-18	39131E-17	50956E+00	69831E+02	-36172E+02	.00000E+00
03	-990030E-18	41263E-17	-31099E+01	76450E+02	-49841E+02	.00000E+00
21	-848130E-18	25855E-17	-59339E+01	19322E+02	-67539E+02	.00000E+00
29	-390050E-18	00000E+00	-61774E+01	.00000E+00	-49734E+02	.00000E+00
26	-197000E-18	42441E-17	-19635E+00	73869E+02	-19589E+02	.00000E+00
19	-896060E-18	43135E-17	-40755E+01	78972E+02	-28277E+02	.00000E+00
18	-360200E-18	58664E-18	-68999E+01	24051E-01	-83520E-01	.00000E+00
17	-129260E-18	00000E+00	-69006E+01	.00000E+00	-66999E-01	.00000E+00
16	-330030E-18	42932E-17	-48148E+00	77821E+02	-51932E+01	.00000E+00
10	-572350E-18	41482E-17	-44579E+01	79189E+02	-63865E+01	.00000E+00
14	-241790E-18	33862E-18	-69018E+01	13350E-01	-69468E-01	.00000E+00
13	-881370E-19	00000E+00	-69022E+01	.00000E+00	-61242E-01	.00000E+00
12	-300970E-18	41677E-17	-48996E+00	79538E+02	28467E+01	.00000E+00
11	-327000E-18	40189E-17	-44884E+01	79084E+02	25728E+01	.00000E+00
10	-143200E-18	26224E-18	-69034E+01	72459E-02	-49548E-01	.00000E+00
9	-706910E-18	00000E+00	-69036E+01	.00000E+00	-45187E-01	.00000E+00
8	-179940E-18	40516E-17	-39063E+00	80447E+02	38761E+01	.00000E+00
7	-140990E-18	39044E-17	-43914E+01	79798E+02	48477E+01	.00000E+00
6	-602670E-19	27351E-18	-69043E+01	44259E-02	-25616E-01	.00000E+00
5	-400190E-19	00000E+00	-69045E+01	.00000E+00	-23654E-01	.00000E+00
4	00000E+00	40077E-17	-33128E+00	81060E+02	.00000E+00	.00000E+00
3	00000E+00	38388E-17	-43195E+01	80293E+02	.00000E+00	.00000E+00
2	00000E+00	27310E-18	-69046E+01	37044E-02	.00000E+00	.00000E+00
1	00000E+00	00000E+00	-69048E+01	.00000E+00	.00000E+00	.00000E+00

TABLE 3.2.4 - EIGENVECTOR (MODE SHAPE) FOR MODEL TYPE II
FOURTH ITERATION

Thickness of plate = 20 mm
 Anchor diameter = 32 mm
 Concrete spring constant = 4.0×10^7 kn/m²m
 Anchor spring constant = 8.0×10^4 kn/m

Eigenvalue/frequency -

Fundamental frequency = 5822 rad/sec
 = 926.6 cycles/sec
 Period = 0.1079×10^{-2} sec

node displacements / rotations						
node number	x-translation	y-translation	z-translation	x-rotation	y-rotation	z-rotation
36	27161E-18	19567E-17	45178E+01	60809E+02	-56450E+02	.00000E+00
35	-67973E-18	15826E-17	15580E+01	55139E+02	-62305E+02	.00000E+00
34	-16290E-17	13634E-17	-32208E+00	21015E+02	-70881E+02	.00000E+00
33	-18787E-17	.00000E+00	-82608E+00	.00000E+00	-73842E+02	.00000E+00
32	27521E-18	26012E-17	31151E+01	63453E+02	-54627E+02	.00000E+00
31	-56620E-18	24789E-17	-22333E-01	60030E+02	-64039E+02	.00000E+00
30	-14509E-17	18922E-17	-21458E+01	23154E+02	-76120E+02	.00000E+00
29	-15446E-17	.00000E+00	-26975E+01	.00000E+00	-76323E+02	.00000E+00
28	20533E-18	32604E-17	18077E+01	67699E+02	-46520E+02	.00000E+00
27	-90122E-18	34155E-17	-16155E+01	68455E+02	-62260E+02	.00000E+00
26	-12017E-17	34575E-17	-40809E+01	23864E+02	-79718E+02	.00000E+00
25	-10540E-17	.00000E+00	-45811E+01	.00000E+00	-72517E+02	.00000E+00
24	40245E-18	38440E-17	70735E+00	73208E+02	-37839E+02	.00000E+00
23	-97833E-18	40567E-17	-30503E+01	78511E+02	-50399E+02	.00000E+00
22	-83748E-18	25564E-17	-59317E+01	19690E+02	-67608E+02	.00000E+00
21	-39647E-18	.00000E+00	-61801E+01	.00000E+00	-49484E+02	.00000E+00
20	-16237E-18	41921E-17	-56921E-01	76025E+02	-22107E+02	.00000E+00
19	-80371E-18	43648E-17	-40335E+01	80442E+02	-28955E+02	.00000E+00
18	-35932E-18	55262E-18	-68996E+01	24297E-01	-83646E-01	.00000E+00
17	-13012E-18	.00000E+00	-69003E+01	.00000E+00	-67007E-01	.00000E+00
16	-30676E-18	42586E-17	-40018E+00	79033E+02	-71815E+01	.00000E+00
15	-56525E-18	41195E-17	-44334E+01	80111E+02	-69534E+01	.00000E+00
14	-24148E-18	33705E-18	-69016E+01	13474E-01	-69599E-01	.00000E+00
13	-89106E-19	.00000E+00	-69019E+01	.00000E+00	-61292E-01	.00000E+00
12	-26563E-18	41473E-17	-44938E+00	80119E+02	16151E+01	.00000E+00
11	-32369E-18	40037E-17	-44766E+01	79579E+02	22041E+01	.00000E+00
10	-14321E-18	28165E-18	-69031E+01	72908E-02	-49656E-01	.00000E+00
9	-71354E-19	.00000E+00	-69033E+01	.00000E+00	-45239E-01	.00000E+00
8	-16899E-18	40397E-17	-37251E+00	80696E+02	33097E+01	.00000E+00
7	-14527E-18	38971E-17	-43869E+01	80044E+02	46739E+01	.00000E+00
6	-66337E-19	27332E-18	-69040E+01	44264E-02	-25677E-01	.00000E+00
5	-48417E-19	.00000E+00	-69042E+01	.00000E+00	-23685E-01	.00000E+00
4	.00000E+00	39987E-17	-32020E+00	81269E+02	.00000E+00	.00000E+00
3	.00000E+00	38342E-17	-43172E+01	80459E+02	.00000E+00	.00000E+00
2	.00000E+00	27301E-18	-69044E+01	36906E-02	.00000E+00	.00000E+00
1	.00000E+00	.00000E+00	-69045E+01	.00000E+00	.00000E+00	.00000E+00

TABLE 3.2.1

COMBINATION OF PLATE THICKNESS, ANCHOR DIAMETERS
AND BASE STIFFNESSES SHOWING THE FIRST PATTERN
OF THE MODE SHAPE (SOME PORTION NEAR RIGHT HAND
CORNER LIFTED-UP)

Base stiffness in kn/m^2	Model Type	Mesh Size in mm	Anchor diameter in mm	Plate Thickness in mm
4.0e+06	I	50x50	25	--
			32	--
		50x25	25	--
			32	--
	II	50x50	25	16
			32	16 20
		50x25	25	--
			32	20
2.0e+07	I	50x50	25	16 20
			32	16 20 24
		50x25	25	20 24
			32	24
	II	50x50	25	16
			32	16 24 28
		50x25	25	20
			32	20 24
4.0e+07	I	50x50	25	16 20 24 28 32
			32	16 20 24 28 32
		50x25	25	24 28
			32	20 24 28
	II	50x50	25	16 24 28 32
			32	16 20 24 28 32
		50x25	25	24 28 32
			32	20 24 28 32
2.0e+08	I	50x50	25	32 40
			32	32 40
		50x25	25	32 40
			32	32 40
	II	50x50	25	24 32 40
			32	32 40
		50x25	25	24 28 32 40
			32	24 28 32 40
4.0e+08	I	50x50	25	--
			32	--
		50x25	25	--
			32	--
	II	50x50	25	--
			32	--
		50x25	25	--
			32	--

TABLE 3 Σ 2

COMBINATION OF PLATE THICKNESS , ANCHOR DIAMETERS
AND BASE STIFFNESSES SHOWING THE SECOND PATTERN
OF THE MODE SHAPE (COMPLETE PLATE UNDER COMPRES-
SION)

Base stiffness in kn/m^2	Model Type	Mesh Size in mm	Anchor diameter in mm	Plate Thickness in mm
4.0e+06	I	50x50	25	16 20 24 28 32
			32	16 20 24 28 32
		50x25	25	20 24 28
			32	20 24 28
	II	50x50	25	20 24 28 32
			32	24 28 32 40
		50x25	25	20 24 28
			32	24 28
2.0e+07	I	50x50	25	32
			32	40
		50x25	25	32 40
			32	40
	II	50x50	25	32 40
			32	40
		50x25	25	28 32 40
			32	28 40
4.0e+07	I	50x50	25	40
			32	40
		50x25	25	32 40
			32	32 40
	II	50x50	25	40
			32	40
		50x25	25	40
			32	40
2.0e+08	I	50x50	25	--
			32	--
		50x25	25	--
			32	--
	II	50x50	25	--
			32	--
		50x25	25	--
			32	--
4.0e+08	I	50x50	25	--
			32	--
		50x25	25	--
			32	--
	II	50x50	25	24
			32	--
		50x25	25	--
			32	--

TABLE 3 3

COMBINATION OF PLATE THICKNESS, ANCHOR DIAMETERS
AND BASE STIFFNESSES SHOWING THE THIRD PATTERN
OF THE MODE SHAPE (COMPLETE PLATE UNDER LIFT-OFF)

Base stiffness in $\text{kn/m}^2/\text{m}$	Model Type	Mesh Size in mm	Anchor diameter in mm	Plate Thickness in mm
4.0e+06	I	50x50	25	40
			32	40
		50x25	25	32 40
			32	32 40
	II	50x50	25	40
			32	--
2.0e+07	I	50x50	25	24 28
			32	32
		50x25	25	28
			32	32
	II	50x50	25	24 28
			32	32
4.0e+07	I	50x50	25	--
			32	--
		50x25	25	--
			32	--
	II	50x50	25	--
			32	--
2.0e+08	I	50x50	25	24
			32	--
		50x25	25	--
			32	--
	II	50x50	25	24
			32	--
4.0e+08	I	50x50	25	24
			32	--
		50x25	25	24
			32	--
	II	50x50	25	32
			32	--
		50x25	25	24
			32	--

TABLE NO. 3 4 :-

EIGENVECTORS(Z-TRANSLATION ONLY)FOR DIFFERENT PLATE
THICKNESS AND ANCHOR DIAMETERS.

$K_c = 4.0E+06 \text{ } Kd/m^2/m$
Mesh size = 50X50
Model type I

node	Thickness of plate in mm											
	16		20		24		28		32		40	
	Anchor diameter in mm											
	32	25	32	25	32	25	32	25	32	25	32	25
20	- 3.001	- 3.001	-1.6715	-1.6715	-2.9783	-2.9783	-3.9227	-3.9227	-4.5450	-4.5450	5.4237	5.724
19	-1.9265	-1.9265	-3.0890	-3.0890	-4.0454	-4.0454	-4.6930	-4.6930	-5.0971	-5.0971	5.6005	5.814
18	-3.0236	-3.0236	-4.0874	-4.0874	-4.8124	-4.8124	-5.2522	-5.2522	-5.5002	-5.5002	5.7784	5.904
17	-3.4106	-3.4106	-4.4989	-4.4989	-5.1480	-5.1480	-5.5040	-5.5040	-5.6846	-5.6846	5.8650	5.949
16	-3.4600	-3.4600	-4.5790	-4.5790	-5.2209	-5.2209	-5.5614	-5.5614	-5.7277	-5.7277	5.8736	5.953
15	-2.0369	-2.0369	-3.1623	-3.1623	-4.0863	-4.0863	-4.7149	-4.7149	-5.1090	-5.1090	5.6367	5.838
14	-4.0336	-4.0336	-4.6852	-4.6852	-5.1717	-5.1717	-5.4756	-5.4756	-5.6448	-5.6448	5.7884	5.901
13	-5.3831	-5.3831	-5.7205	-5.7205	-5.9123	-5.9123	-5.9956	-5.9956	-6.0114	-6.0114	5.9538	5.997
12	-5.7073	-5.7073	-5.9981	-5.9981	-6.1213	-6.1213	-6.1465	-6.1465	-6.1197	-6.1197	6.0042	6.011
11	-5.7233	-5.7233	-6.0247	-6.0247	-6.1456	-6.1456	-6.1658	-6.1658	-6.1342	-6.1342	5.9889	6.011
10	-3.2275	-3.2275	-4.2051	-4.2051	-4.8697	-4.8697	-5.2780	-5.2780	-5.5110	-5.5110	5.8416	5.931
9	-5.4546	-5.4546	-5.7628	-5.7628	-5.9338	-5.9338	-6.0060	-6.0060	-6.0163	-6.0163	5.9761	6.001
8	-6.8533	-6.8533	-6.7228	-6.7228	-6.5835	-6.5835	-6.4481	-6.4481	-6.3221	-6.3221	6.0821	6.051
7	-6.8540	-6.8540	-6.7237	-6.7237	-6.5845	-6.5845	-6.4492	-6.4492	-6.3232	-6.3232	6.0828	6.051
6	-6.8543	-6.8543	-6.7240	-6.7240	-6.5849	-6.5849	-6.4496	-6.4496	-6.3236	-6.3236	6.0831	6.051
5	-3.6039	-3.6039	-4.5466	-4.5466	-5.1302	-5.1302	-5.4667	-5.4667	-5.6463	-5.6463	5.9274	5.971
4	-5.7838	-5.7838	-6.0231	-6.0231	-6.1218	-6.1218	-6.1384	-6.1384	-6.1097	-6.1097	6.0343	6.031
3	-6.8535	-6.8535	-6.7230	-6.7230	-6.5837	-6.5837	-6.4484	-6.4484	-6.3224	-6.3224	6.0823	6.051
2	-6.8541	-6.8541	-6.7237	-6.7237	-6.5846	-6.5846	-6.4493	-6.4493	-6.3233	-6.3233	6.0829	6.051
1	-6.8543	-6.8543	-6.7240	-6.7240	-6.5849	-6.5849	-6.4496	-6.4496	-6.3236	-6.3236	6.0831	6.051

TABLE NO. 3.5 -

EIGENVECTORS (Z-TRANSLATION ONLY) FOR DIFFERENT COMBINATION
OF PLATE THICKNESS AND ANCHOR DIAMETERS.

$K_c = 2.0E+07 \text{ KN/m}^2/\text{m}$

Mesh size = 50X50

Model type I

node	Thickness of plate in mm												
	16		20		24		28		32		40		
	Anchor diameter in mm												
	32	25	32	25	32	25	32	25	32	25	32	25	
20	3.6622	3.6622	3.7796	1.7595	.6102	4.8045			5.3133	4.9546	-1.3172	-2.8571	-2.8571
19	1.1693	1.1693	1.2366	-.2550	-1.1881	5.2570			5.5972	5.3141	-2.7617	-3.8777	-3.8777
18	-.3399	-.3399	-.3149	-1.4430	-2.3521	5.6982			5.8768	5.6697	-3.7720	-4.6114	-4.6114
17	-.6563	-.6563	-.6501	-1.7201	-2.7212	5.9075			6.0112	5.8399	-4.1816	-4.9345	-4.9345
16	-.6103	-.6103	-.6084	-1.6984	-2.7516	5.9269			6.0246	5.8558	-4.2568	-5.0056	-5.0056
15	1.0822	1.0822	1.1491	-.3546	-1.3014	5.3508			5.6541	5.3695	-2.8380	-3.9125	-3.9125
14	-2.0476	-2.0476	-2.0000	-2.9602	-3.5451	5.7483			5.9018	5.7017	-4.4111	-4.9431	-4.9431
13	-4.1959	-4.1959	-4.1596	-4.7142	-5.0554	6.1622			6.1632	6.0340	-5.4779	-5.6454	-5.6454
12	-4.6202	-4.6202	-4.5904	-5.0732	-5.4012	6.2844			6.2414	6.1331	-5.7615	-5.8465	-5.8465
11	-4.5964	-4.5964	-4.5685	-5.0633	-5.4108	6.2466			6.2182	6.1021	-5.7879	-5.8712	-5.8712
10	-.5354	-.5354	-.5095	-1.6705	-2.5746	5.8628			5.9762	5.8019	-3.8967	-4.6577	-4.6577
9	-4.2708	-4.2708	-4.2339	-4.7963	-5.1334	6.2201			6.1982	6.0805	-5.5217	-5.6622	-5.6622
8	-6.9403	-6.9403	-6.8921	-6.8578	-6.7579	6.4944			6.3697	6.2970	-6.5216	-6.2730	-6.2730
7	-6.9420	-6.9420	-6.8953	-6.8600	-6.7607	6.4948			6.3701	6.2978	-6.5254	-6.2774	-6.2774
6	-6.9426	-6.9426	-6.8965	-6.8609	-6.7618	6.4950			6.3703	6.2980	-6.5268	-6.2790	-6.2790
5	-.9688	-.9688	-.9563	-2.0425	-2.9648	6.0741			6.1098	5.9733	-4.2415	-4.9054	-4.9054
4	-4.7524	-4.7524	-4.7196	-5.1976	-5.4946	6.3639			6.2891	6.1969	-5.7884	-5.8400	-5.8400
3	-6.9407	-6.9407	-6.8927	-6.8583	-6.7585	6.4945			6.3699	6.2972	-6.5224	-6.2740	-6.2740
2	-6.9421	-6.9421	-6.8955	-6.8602	-6.7609	6.4949			6.3702	6.2978	-6.5256	-6.2776	-6.2776
1	-6.9427	-6.9427	-6.8966	-6.8609	-6.7618	6.4950			6.3703	6.2981	-6.5269	-6.2791	-6.2791

TABLE NO. 3.6.11

EIGENVECTORS , ONLY Z-TRANSLATIONS FOR
 EIGENVECTOS (Z-TRANSLATION ONLY) FOR DIFFERENT COMBINATION
 $KC = 4.0E+07 \text{ KN/m}^2/\text{m}$
 Mesh size = 50X50
 Model type I

node	Thickness of plate in mm											
	16		20		24		28		32		40	
	Anchor diameter in mm											
	32	25	32	25	32	25	32	25	32	25	32	25
20	7.2005	7.2005	3.7796		1.9337	1.9337	1.0883	1.0883	.0549	.0549	-1.3276	-1.2060
19	3.8233	3.8233	1.2366		-.0930	-.0930	-.7964	-.7964	-1.5930	-1.5930	-2.7393	-2.6403
18	1.6399	1.6399	-.3149		-1.2566	-1.2566	-1.9792	-1.9792	-2.6876	-2.6876	-3.7277	-3.6592
17	1.0018	1.0018	-.6501		-1.5016	-1.5016	-2.3229	-2.3229	-3.0607	-3.0607	-4.1311	-4.0723
16	.9772	.9772	-.6084		-1.4668	-1.4668	-2.3373	-2.3373	-3.1034	-3.1034	-4.2083	-4.1447
15	3.3408	3.3408	1.1491		-.1838	-.1838	-.9063	-.9063	-1.6999	-1.6999	-2.8105	-2.7254
14	-.6068	-.6068	-2.0000		-2.8225	-2.8225	-3.2812	-3.2812	-3.7365	-3.7365	-4.3408	-4.2783
13	-3.2962	-3.2962	-4.1596		-4.5997	-4.5997	-4.8773	-4.8773	-5.1082	-5.1082	-5.3784	-5.3525
12	-3.8502	-3.8502	-4.5904		-4.9543	-4.9543	-5.2308	-5.2308	-5.4345	-5.4345	-5.6572	-5.6390
11	-3.8344	-3.8344	-4.5685		-4.9399	-4.9399	-5.2351	-5.2351	-5.4495	-5.4495	-5.6846	-5.6559
10	.7897	.7897	-.5095		-1.4737	-1.4737	-2.2052	-2.2052	-2.8890	-2.8890	-3.8426	-3.8016
9	-3.5482	-3.5482	-4.2339		-4.6783	-4.6783	-4.9567	-4.9567	-5.1777	-5.1777	-5.4179	-5.4017
8	-6.9460	-6.9460	-6.8921		-6.8126	-6.8126	-6.7205	-6.7205	-6.6138	-6.6138	-6.3851	-6.3931
7	-6.9484	-6.9484	-6.8953		-6.8167	-6.8167	-6.7255	-6.7255	-6.6196	-6.6196	-6.3922	-6.4002
6	-6.9493	-6.9493	-6.8965		-6.8182	-6.8182	-6.7273	-6.7273	-6.6217	-6.6217	-6.3949	-6.4030
5	.0285	.0285	-.9563		-1.8288	-1.8288	-2.5934	-2.5934	-3.2604	-3.2604	-4.1788	-4.1582
4	-4.1805	-4.1805	-4.7196		-5.0797	-5.0797	-5.3327	-5.3327	-5.5092	-5.5092	-5.6770	-5.6759
3	-6.9464	-6.9464	-6.8927		-6.8135	-6.8135	-6.7216	-6.7216	-6.6151	-6.6151	-6.3867	-6.3947
2	-6.9485	-6.9485	-6.8955		-6.8170	-6.8170	-6.7258	-6.7258	-6.6199	-6.6199	-6.3926	-6.4006
1	-6.9494	-6.9494	-6.8966		-6.8183	-6.8183	-6.7275	-6.7275	-6.6218	-6.6218	-6.3950	-6.4031

TABLE NO 37

EIGENVECTOS (Z-TRANSLATION ONLY) FOR DIFFERENT COMBINATION
OF PLATE THICKNESS AND ANCHOR DIAMETERS.

$K_c = 2.0E+08 \text{ kN/m}^2/\text{m}$
Mesh size = 50X50
Model type I

node	Thickness of plate in mm											
	16		20		24		28		32		40	
	Anchor diameter in mm											
	32	25	32	25	32	25	32	25	32	25	32	25
20					4.8045				4.8865	4.8865	1.9224	1.9224
19					5.2570				1.9061	1.9061	-1.0804	-1.0804
18					5.6982				-0.0121	-0.0121	-1.2275	-1.2275
17					5.9075				-0.4792	-0.4792	-1.4739	-1.4739
16					5.9269				-0.4575	-0.4575	-1.4435	-1.4435
15					5.3508				1.8216	1.8216	-1.1654	-1.1654
14					5.7483				-1.5881	-1.5881	-2.7412	-2.7412
13					6.1622				-3.9256	-3.9256	-4.4740	-4.4740
12					6.2844				-4.4156	-4.4156	-4.8285	-4.8285
11					6.2466				-4.4009	-4.4009	-4.8194	-4.8194
10					5.8628				-1.1969	-1.1969	-1.4304	-1.4304
9					6.2201				-3.9954	-3.9954	-4.5437	-4.5437
8					6.4944				-6.7508	-6.7508	-6.6080	-6.6080
7					6.4948				-6.7646	-6.7646	-6.6258	-6.6258
6					6.4950				-6.7698	-6.7698	-6.6324	-6.6324
5					6.0741				-1.7522	-1.7522	-1.7775	-1.7775
4					6.3639				-4.5303	-4.5303	-4.9343	-4.9343
3					6.4945				-6.7535	-6.7535	-6.6117	-6.6117
2					6.4949				-6.7654	-6.7654	-6.6269	-6.6269
1					6.4950				-6.7701	-6.7701	-6.6329	-6.6329

TABLE NO. 3.2.1

EIGENVECTORS (X-TRANSLATION ONLY) FOR DIFFERENT COMBINATION
OF PLATE THICKNESS AND ANCHOR DIAMETERS.

$K_c = 4.0E+06 \text{ KN/m}^2/\text{m}$
Mesh size = 50X50
Model type I

node	Thickness of plate in mm											
	16		20		24		28		32		40	
	Anchor diameter in mm											
	32	25	32	25	32	25	32	25	32	25	32	25
20	2.6740	1.4356	.3982	- .7706	-1.2843	-2.0709	-2.5991	-3.2306	-3.5538	-4.0347	-4.6314	5.7244
19	.3256	-.6308	-1.4255	-2.2988	-2.7306	-3.3447	-3.6974	-4.1742	-4.3660	-4.7217	-5.0763	5.8142
18	-1.4663	-2.1507	-2.8916	-3.4678	-3.9127	-4.3396	-4.6015	-4.9166	-5.0381	-5.2646	-5.4473	5.9048
17	-1.9565	-2.6236	-3.4256	-3.9458	-4.4023	-4.7645	-4.9992	-5.2480	-5.3433	-5.5129	-5.6211	5.9492
16	-1.8233	-2.5825	-3.4177	-3.9762	-4.4437	-4.8243	-5.0497	-5.3036	-5.3884	-5.5579	-5.6500	5.9538
15	-.1226	-.8997	-1.7395	-2.4958	-2.9382	-3.4688	-3.8301	-4.2510	-4.4523	-4.7703	-5.1165	5.8321
14	-2.6383	-3.2123	-3.6537	-4.1724	-4.3808	-4.7481	-4.8890	-5.1689	-5.2197	-5.4260	-5.5274	5.9091
13	-4.7293	-5.0124	-5.2440	-5.4782	-5.5719	-5.7335	-5.7606	-5.8746	-5.8514	-5.9300	-5.8670	5.9930
12	-5.1406	-5.4038	-5.6221	-5.8039	-5.8862	-6.0024	-6.0038	-6.0755	-6.0335	-6.0772	-5.9685	6.0190
11	-4.8546	-5.2597	-5.4818	-5.7424	-5.8164	-5.9796	-5.9672	-6.0670	-6.0127	-6.0742	-5.9603	6.0110
10	-2.3440	-2.6893	-3.4845	-3.8510	-4.2859	-4.5541	-4.8320	-5.0434	-5.1840	-5.3421	-5.5128	5.9350
9	-5.0405	-5.2048	-5.4523	-5.6082	-5.7033	-5.8099	-5.8423	-5.9203	-5.9035	-5.9583	-5.8906	6.0040
8	-6.9058	-6.8868	-6.7908	-6.7593	-6.6569	-6.6242	-6.5180	-6.4856	-6.3833	-6.3542	-6.1423	6.0580
7	-6.9066	-6.8876	-6.7918	-6.7603	-6.6584	-6.6255	-6.5196	-6.4870	-6.3850	-6.3556	-6.1440	6.0580
6	-6.9069	-6.8879	-6.7922	-6.7607	-6.6589	-6.6259	-6.5202	-6.4875	-6.3856	-6.3561	-6.1447	6.0580
5	-3.1559	-3.3227	-4.1362	-4.3410	-4.7905	-4.9456	-5.2077	-5.3301	-5.4590	-5.5494	-5.6623	5.9790
4	-5.6764	-5.7156	-5.9384	-5.9807	-6.0652	-6.0913	-6.1055	-6.1214	-6.0936	-6.1016	-5.9926	6.0330
3	-6.9060	-6.8870	-6.7909	-6.7596	-6.6574	-6.6245	-6.5185	-6.4859	-6.3837	-6.3545	-6.1427	6.0580
2	-6.9067	-6.8877	-6.7919	-6.7604	-6.6586	-6.6255	-6.5198	-6.4871	-6.3851	-6.3557	-6.1441	6.0580
1	-6.9070	-6.8880	-6.7923	-6.7608	-6.6590	-6.6260	-6.5203	-6.4875	-6.3857	-6.3562	-6.1447	6.0580

TABLE NO. 3.10:-

EIGENVECTORS (Z-TRANSLATION ONLY) FOR DIFFERENT COMBINATION
OF PLATE THICKNESS AND ANCHOR DIAMETERS.

$K_c = 2.0E+07 \text{ KN/m}^2/\text{m}$
Mesh size = 50X50
Model type II

node	Thickness of plate in mm											
	16		20		24		28		32		40	
	Anchor diameter in mm											
	32	25	32	25	32	25	32	25	32	25	32	25
20	2.6740	4.6244			1.4352	4.8045	.0932	5.3133	4.9546	-1.0734	-2.4948	-2.6736
19	3236	1.7882			-1.5794	5.2570	-1.6023	5.5972	5.3141	-2.5628	-3.6004	-3.7374
18	-1.4663	-0.6662			-1.9536	5.6962	-2.6222	5.8768	5.6697	-3.6323	-4.4305	-4.5202
17	-1.9565	-4.441			-2.3548	5.9075	-3.2514	6.0112	5.6399	-4.0601	-4.7931	-4.8636
16	-1.8233	-3301			-2.3267	5.9269	-3.2760	6.0246	5.6558	-4.1273	-4.8596	-4.9326
15	-1.1226	1.6732			-1.7628	5.3508	-1.7680	5.6541	5.3895	-2.6664	-3.6688	-3.7890
14	-2.6383	-1.6664			-3.1439	5.7483	-3.7380	5.9018	5.7017	-4.2846	-4.7772	-4.8591
13	-4.7293	-4.0589			-4.8612	6.1622	-5.1741	6.1632	6.0340	-5.4236	-5.5818	-5.6134
12	-5.1406	-4.4981			-5.2549	6.2644	-5.5158	6.2414	6.1331	-5.7218	-5.8089	-5.8278
11	-4.8546	-4.3433			-5.1701	6.2466	-5.4744	6.2182	6.1021	-5.7275	-5.6142	-5.6428
10	-2.3440	-3369			-2.3335	5.8628	-3.1374	5.9762	5.8019	-3.8118	-4.5402	-4.5980
9	-5.0405	-4.1684			-5.0176	6.2201	-5.2852	6.1982	6.0805	-5.4867	-5.6208	-5.6412
8	-6.9058	-6.9429			-6.7786	6.4944	-6.6668	6.3697	6.2970	-6.5348	-6.2981	-6.2858
7	-6.9066	-6.9446			-6.7816	6.4948	-6.6704	6.3701	6.2978	-6.5388	-6.3030	-6.2904
6	-6.9069	-6.9452			-6.7827	6.4950	-6.6717	6.3703	6.2980	-6.5403	-6.3047	-6.2921
5	-3.1559	-9254			-2.8540	6.0741	-3.6038	6.1098	5.9733	-4.1964	-4.8434	-4.8739
4	-5.6764	-4.7422			-5.4722	6.3639	-5.6677	6.2891	6.1969	-5.7829	-5.8379	-5.8390
3	-6.9060	-6.9433			-6.7794	6.4945	-6.6677	6.3699	6.2972	-6.5358	-6.2993	-6.2869
2	-6.9067	-6.9447			-6.7818	6.4949	-6.6707	6.3702	6.2978	-6.5391	-6.3032	-6.2907
1	-6.9070	-6.9453			-6.7828	6.4950	-6.6718	6.3703	6.2981	-6.5404	-6.3048	-6.2922

TABLE NO. 3.11 -

EIGENVECTORS (Z-TRANSLATION ONLY) FOR DIFFERENT COMBINATION
OF PLATE THICKNESS AND ANCHOR DIAMETERS.

$K_c = 4.0E+07 \text{ KN/m}^2/\text{m}$

Mesh size = 50X50

Model type II

node	Thickness of plate in mm											
	16		20		24		28		32		40	
	Anchor diameter in mm											
	32	25	32	25	32	25	32	25	32	25	32	25
20	7.8376	7.5903	4.7881		2.7005	2.2093	1.5318	1.3173	.4914	.2786	-1.0872	-1.2060
19	4.4852	4.2012	1.8794		.4452	.1157	-.4715	-.6288	-1.2746	-1.4300	-2.5436	-2.6403
18	2.3868	2.0600	-.0351		-.9668	-1.1315	-1.7702	-1.8721	-2.4846	-2.5842	-3.5919	-3.6592
17	2.0632	1.5830	-.4368		-1.2741	-1.3888	-2.1264	-2.2233	-2.8858	-2.9724	-4.0140	-4.0723
16	2.2709	1.6816	-.3267		-1.1992	-1.3291	-2.1044	-2.2196	-2.9076	-3.0048	-4.0813	-4.1447
15	3.6113	3.5137	1.7654		.2857	-.0177	-.6168	-.7566	-1.4172	-1.5548	-2.6425	-2.7254
14	-.3624	-.4583	-1.6094		-2.4884	-2.6840	-3.0600	-3.1670	-3.5332	-3.6323	-4.2173	-4.2783
13	-3.0683	-3.1670	-4.0207		-4.4667	-4.5401	-4.7832	-4.8290	-5.0240	-5.0653	-5.3270	-5.3525
12	-3.4818	-3.6492	-4.4675		-4.8501	-4.9016	-5.1493	-5.1896	-5.3706	-5.4024	-5.6207	-5.6390
11	-3.2387	-3.5110	-4.3125		-4.7525	-4.8434	-5.0946	-5.1641	-5.3458	-5.3974	-5.6273	-5.6559
10	.8213	.8160	-.3059		-1.2971	-1.4130	-2.0800	-2.1404	-2.7624	-2.8239	-3.7618	-3.8016
9	-3.5268	-3.5327	-4.1302		-4.5879	-4.6421	-4.8943	-4.9244	-5.1215	-5.1489	-5.3859	-5.4017
8	-6.9420	-6.9440	-6.8948		-6.8225	-6.8175	-6.7325	-6.7268	-6.6292	-6.6218	-6.4007	-6.3931
7	-6.9443	-6.9463	-6.8981		-6.8267	-6.8218	-6.7378	-6.7318	-6.6332	-6.6277	-6.4082	-6.4003
6	-6.9451	-6.9472	-6.8993		-6.8282	-6.8231	-6.7395	-6.7337	-6.6374	-6.6299	-6.4109	-6.4030
5	-.0099	.0089	-.9135		-1.7717	-1.8128	-2.5411	-2.5663	-3.2015	-3.2301	-4.1382	-4.1582
4	-4.1993	-4.1906	-4.7103		-5.0683	-5.0771	-5.3233	-5.3278	-5.5008	-5.5049	-5.6748	-5.6759
3	-6.9425	-6.9445	-6.8956		-6.8234	-6.8184	-6.7337	-6.7279	-6.6306	-6.6231	-6.4025	-6.3947
2	-6.9445	-6.9465	-6.8983		-6.8270	-6.8219	-6.7380	-6.7322	-6.6356	-6.6281	-6.4086	-6.4008
1	-6.9453	-6.9473	-6.8994		-6.8283	-6.8232	-6.7396	-6.7338	-6.6376	-6.6300	-6.4111	-6.4032

TABLE NO. 3 12 -

EIGENVECTORS (Z-TRANSLATION ONLY) FOR DIFFERENT COMBINATION
OF PLATE THICKNESS AND ANCHOR DIAMETERS.

Kc = 2.0E+08 KN/m²/m
Mesh size = 50x50
Model type II

node	Thickness of plate in mm																																			
	16						20						24						28						32						40					
	Anchor diameter in mm																																			
	32	25	32	25	32	25	32	25	32	25	32	25	32	25	32	25	32	25	32	25	32	25														
20							4.8045						4.8865	5.0582	2.0188	1.9224																				
19							5.2570						1.9061	2.0212	-0.0115	-0.0804																				
18							5.6982						-0.0121	0.0458	-1.1855	-1.2275																				
17							5.9075						-0.4792	-0.4468	-1.4308	-1.4739																				
16							5.9269						-0.4575	-0.4234	-1.3879	-1.4435																				
15							5.3508						1.8216	1.9170	-1.1026	-1.1654																				
14							5.7483						-1.5881	-1.5277	-2.6895	-2.7412																				
13							6.1622						-3.9256	-3.9018	-4.4531	-4.4740																				
12							6.2844						-4.4156	-4.3987	-4.8083	-4.8285																				
11							6.2466						-4.4009	-4.3698	-4.7799	-4.8194																				
10							5.8628						-0.1969	-0.1707	-1.4059	-1.4304																				
9							6.2201						-3.9954	-3.9818	-4.5300	-4.5437																				
8							6.4944						-6.7508	-6.7507	-6.6110	-6.6080																				
7							6.4948						-6.7646	-6.7645	-6.6288	-6.6258																				
6							6.4950						-6.7698	-6.7696	-6.6355	-6.6324																				
5							6.0741						-0.7522	-0.7504	-1.7704	-1.7775																				
4							6.3639						-4.5303	-4.5306	-4.9338	-4.9343																				
3							6.4945						-6.7535	-6.7535	-6.6148	-6.6117																				
2							6.4949						-6.7654	-6.7654	-6.6300	-6.6269																				
1							6.4950						-6.7701	-6.7700	-6.6360	-6.6329																				

TABLE NO. 3.14

EIGENVECTORS (Z-TRANSLATION ONLY) FOR DIFFERENT COMBINATION
OF PLATE THICKNESS AND ANCHOR DIAMETERS.

KC = 4.0E+06 KN/m²/m
Mesh size = 50X25
Model type I

node num	Thickness of plate in mm											
	16		20		24		28		32		40	
	Anchor diameter in mm											
	32	25	32	25	32	25	32	25	32	25	32	25
36	-1.9987	-1.9987	-3.3274	-3.3274	-4.2225	-4.2225	4.9203	5.5459	5.4045	5.7143		
35	-3.5124	-3.5124	-4.4104	-4.4104	-4.9769	-4.9769	5.3576	5.7706	5.6188	5.8227		
34	-4.5263	-4.5263	-5.1424	-5.1424	-5.4890	-5.4890	5.7613	5.9789	5.8198	5.9246		
33	-4.8386	-4.8386	-5.3715	-5.3715	-5.6506	-5.6506	5.9245	6.0634	5.9018	5.9663		
32	-2.6102	-2.6102	-3.7605	-3.7605	-4.5227	-4.5227	5.1002	5.6394	5.4934	5.7595		
31	-4.1518	-4.1518	-4.8451	-4.8451	-5.2715	-5.2715	5.5112	5.8503	5.6939	5.8609		
30	-5.1802	-5.1802	-5.5708	-5.5708	-5.7732	-5.7732	5.9044	6.0533	5.8894	5.9601		
29	-5.4726	-5.4726	-5.7802	-5.7802	-5.9191	-5.9191	6.0493	6.1283	5.9622	5.9971		
28	-3.1924	-3.1924	-4.1763	-4.1763	-4.8120	-4.8120	5.2919	5.7390	5.5886	5.8080		
27	-4.7855	-4.7855	-5.2762	-5.2762	-5.5638	-5.5638	5.6809	5.9384	5.7773	5.9033		
26	-5.8443	-5.8443	-6.0055	-6.0055	-6.0614	-6.0614	6.0626	6.1355	5.9668	5.9995		
25	-6.0778	-6.0778	-6.1694	-6.1694	-6.1744	-6.1744	6.1741	6.1932	6.0228	6.0279		
24	-3.6999	-3.6999	-4.5429	-4.5429	-5.0686	-5.0686	5.4962	5.8454	5.6909	5.8601		
23	-5.3388	-5.3388	-5.6534	-5.6534	-5.8197	-5.8197	5.8768	6.0403	5.8750	5.9531		
22	-6.4277	-6.4277	-6.3854	-6.3854	-6.3125	-6.3125	6.2170	6.2157	6.0428	6.0381		
21	-6.5340	-6.5340	-6.4590	-6.4590	-6.3629	-6.3629	6.2656	6.2409	6.0673	6.0506		
20	-4.0905	-4.0905	-4.8296	-4.8296	-5.2708	-5.2708	5.6763	5.9394	5.7817	5.9064		
19	-5.7235	-5.7235	-5.9178	-5.9178	-5.9999	-5.9999	6.0350	6.1227	5.9544	5.9935		
18	-6.7124	-6.7124	-6.5703	-6.5703	-6.4346	-6.4346	6.2982	6.2579	6.0830	6.0586		
17	-6.7126	-6.7126	-6.5705	-6.5705	-6.4349	-6.4349	6.2985	6.2581	6.0832	6.0587		
16	-4.3354	-4.3354	-5.0149	-5.0149	-5.4036	-5.4036	5.7959	6.0022	5.8425	5.9375		
15	-5.9113	-5.9113	-6.0503	-6.0503	-6.0916	-6.0916	6.1165	6.1653	5.9957	6.0146		
14	-6.7129	-6.7129	-6.5708	-6.5708	-6.4352	-6.4352	6.2987	6.2582	6.0834	6.0588		
13	-6.7130	-6.7130	-6.5710	-6.5710	-6.4353	-6.4353	6.2988	6.2583	6.0836	6.0589		
12	-4.4659	-4.4659	-5.1184	-5.1184	-5.4794	-5.4794	5.8525	6.0324	5.8717	5.9526		
11	-5.9791	-5.9791	-6.1011	-6.1011	-6.1279	-6.1279	6.1372	6.1766	6.0066	6.0202		
10	-6.7133	-6.7133	-6.5712	-6.5712	-6.4356	-6.4356	6.2991	6.2584	6.0838	6.0590		
9	-6.7133	-6.7133	-6.5713	-6.5713	-6.4357	-6.4357	6.2991	6.2584	6.0838	6.0590		
8	-4.5226	-4.5226	-5.1663	-5.1663	-5.5155	-5.5155	5.8647	6.0395	5.8783	5.9561		
7	-5.9990	-5.9990	-6.1179	-6.1179	-6.1406	-6.1406	6.1223	6.1694	5.9994	6.0167		
6	-6.7135	-6.7135	-6.5715	-6.5715	-6.4359	-6.4359	6.2993	6.2585	6.0840	6.0591		
5	-6.7135	-6.7135	-6.5715	-6.5715	-6.4359	-6.4359	6.2993	6.2585	6.0840	6.0591		
4	-4.5377	-4.5377	-5.1798	-5.1798	-5.5259	-5.5259	5.8617	6.0383	5.8770	5.9556		
3	-6.0027	-6.0027	-6.1216	-6.1216	-6.1435	-6.1435	6.1051	6.1608	5.9908	6.0125		
2	-6.7136	-6.7136	-6.5716	-6.5716	-6.4360	-6.4360	6.2993	6.2585	6.0840	6.0591		
1	-6.7136	-6.7136	-6.5716	-6.5716	-6.4360	-6.4360	6.2994	6.2586	6.0841	6.0592		

TABLE NO 3 15 -

EIGENVECTORS (Z-TRANSLATION ONLY) FOR DIFFERENT COMBINATION
OF PLATE THICKNESS AND ANCHOR DIAMETERS.

$K_c = 2.0E+07 \text{ KN/m}^2/\text{m}$
Mesh size = 50X25
Model type I

node num	Thickness of plate in mm											
	16		20		24		28		32		40	
	Anchor diameter in mm											
	32	25	32	25	32	25	32	25	32	25	32	25
36												
35												
34												
33												
32												
31												
30												
29												
28												
27												
26												
25												
24												
23												
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18												
17												
16												
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14												
13												
12												
11												
10												
9												
8												
7												
6												
5												
4												
3												
2												
1												

TABLE NO 3 16 -

EIGENVECTORS (Z-TRANSLATION ONLY) FOR DIFFERENT COMBINATION
OF PLATE THICKNESS AND ANCHOR DIAMETERS.

$K_c = 4.0E+07 \text{ KN/m}^2/\text{m}$
Mesh size = 50X25
Model type I

node num	Thickness of plate in mm																							
	16				20				24				28				32				40			
	Anchor diameter in mm																							
	32		25		32		25		32		25		32		25		32		25					
36			3.6209			1.9959	1.9959	.8788	.8788	-1.1027	-1.1027	-1.9838	-1.9838	-3.1659	-3.1659	-4.1805	-4.1805							
35			1.0259			-.2406	-.2406	-1.1715	-1.1715	-3.2089	-3.2089	-4.1805	-4.1805	-4.4915	-4.4915	-4.8041	-4.8041							
34			-.5097			-1.5861	-1.5861	-2.4716	-2.4716	-3.5675	-3.5675	-4.8356	-4.8356	-5.1276	-5.1276	-5.4361	-5.4361							
33			-.8819			-1.9350	-1.9350	-2.8361	-2.8361	-3.8779	-3.8779	-4.8356	-4.8356	-5.1276	-5.1276	-5.4361	-5.4361							
32			2.3421			-.5123	.9457	-.0250	-.0250	-2.8898	-2.8898	-4.2180	-4.2180	-4.8356	-4.8356	-5.1276	-5.1276							
31			-.4881			-3.1006	-3.1006	-3.7230	-3.7230	-4.5795	-4.5795	-5.1276	-5.1276	-5.4361	-5.4361	-5.7335	-5.7335							
30			-2.2973			-3.5007	-3.5007	-4.1104	-4.1104	-4.5795	-4.5795	-5.1276	-5.1276	-5.4361	-5.4361	-5.7335	-5.7335							
29			-2.7368			.0032	-3.5007	-4.1104	-4.1104	-1.5998	-1.5998	-3.7858	-3.7858	-4.4361	-4.4361	-5.1276	-5.1276							
28			1.1806			-2.7694	.0032	-.8441	-.8441	-3.7858	-3.7858	-4.4361	-4.4361	-5.1276	-5.1276	-5.4361	-5.4361							
27			-1.9677			-2.7694	-2.7694	-3.3262	-3.3262	-5.2438	-5.2438	-5.5500	-5.5500	-5.7335	-5.7335	-5.9995	-5.9995							
26			-4.1793			-4.6614	-4.6614	-4.9995	-4.9995	-5.2438	-5.2438	-5.5500	-5.5500	-5.7335	-5.7335	-5.9995	-5.9995							
25			-4.5970			-5.0322	-5.0322	-5.3408	-5.3408	-5.5500	-5.5500	-5.7335	-5.7335	-5.9995	-5.9995	-6.2882	-6.2882							
24			.2535			-.7365	-.7365	-1.5118	-1.5118	-2.2099	-2.2099	-3.3296	-3.3296	-3.9874	-3.9874	-4.6817	-4.6817							
23			-3.2968			-3.8584	-3.8584	-4.2523	-4.2523	-4.5650	-4.5650	-4.8748	-4.8748	-5.1892	-5.1892	-5.4995	-5.4995							
22			-5.9654			-6.0907	-6.0907	-6.1452	-6.1452	-6.3001	-6.3001	-6.4548	-6.4548	-6.6092	-6.6092	-6.7637	-6.7637							
21			-6.1795			-6.2767	-6.2767	-6.3111	-6.3111	-6.4548	-6.4548	-6.6092	-6.6092	-6.7637	-6.7637	-6.9182	-6.9182							
20			-.3566			-1.2281	-1.2281	-1.9825	-1.9825	-2.6594	-2.6594	-3.3296	-3.3296	-3.9874	-3.9874	-4.6817	-4.6817							
19			-4.1751			-4.5865	-4.5865	-4.8773	-4.8773	-5.0976	-5.0976	-5.3079	-5.3079	-5.5182	-5.5182	-5.7285	-5.7285							
18			-6.8952			-6.8124	-6.8124	-6.7144	-6.7144	-6.6040	-6.6040	-6.4936	-6.4936	-6.3832	-6.3832	-6.2728	-6.2728							
17			-6.8958			-6.8132	-6.8132	-6.7155	-6.7155	-6.6053	-6.6053	-6.4949	-6.4949	-6.3845	-6.3845	-6.2741	-6.2741							
16			-.6213			-1.4533	-1.4533	-5.1526	-5.1526	-5.3428	-5.3428	-5.5330	-5.5330	-5.7232	-5.7232	-5.9134	-5.9134							
15			-4.5349			-4.8924	-4.8924	-6.7173	-6.7173	-6.6072	-6.6072	-6.4968	-6.4968	-6.3864	-6.3864	-6.2760	-6.2760							
14			-6.8972			-6.8146	-6.8146	-6.7173	-6.7173	-6.6079	-6.6079	-6.4975	-6.4975	-6.3871	-6.3871	-6.2767	-6.2767							
13			-6.8975			-6.8153	-6.8153	-6.7179	-6.7179	-6.6079	-6.6079	-6.4975	-6.4975	-6.3871	-6.3871	-6.2767	-6.2767							
12			-.6727			-1.5089	-1.5089	-2.3183	-2.3183	-3.0314	-3.0314	-3.7445	-3.7445	-4.4576	-4.4576	-5.1707	-5.1707							
11			-4.6040			-4.9597	-4.9597	-5.2259	-5.2259	-5.4182	-5.4182	-5.6101	-5.6101	-5.8020	-5.8020	-6.0000	-6.0000							
10			-6.8987			-6.8167	-6.8167	-6.7195	-6.7195	-6.6098	-6.6098	-6.4994	-6.4994	-6.3890	-6.3890	-6.2786	-6.2786							
9			-6.8989			-6.8169	-6.8169	-6.7198	-6.7198	-6.6101	-6.6101	-6.5000	-6.5000	-6.3896	-6.3896	-6.2792	-6.2792							
8			-.6429			-1.4944	-1.4944	-2.3333	-2.3333	-3.0686	-3.0686	-3.8039	-3.8039	-4.5392	-4.5392	-5.2745	-5.2745							
7			-4.5896			-4.9545	-4.9545	-5.2321	-5.2321	-5.4326	-5.4326	-5.6331	-5.6331	-5.8336	-5.8336	-6.0341	-6.0341							
6			-6.8997			-6.8179	-6.8179	-6.7209	-6.7209	-6.6113	-6.6113	-6.5010	-6.5010	-6.3906	-6.3906	-6.2802	-6.2802							
5			-6.8998			-6.8180	-6.8180	-6.7211	-6.7211	-6.6116	-6.6116	-6.5013	-6.5013	-6.3909	-6.3909	-6.2805	-6.2805							
4			-.6229			-1.4815	-1.4815	-2.3314	-2.3314	-3.0753	-3.0753	-3.8106	-3.8106	-4.5459	-4.5459	-5.2812	-5.2812							
3			-4.5776			-4.9467	-4.9467	-5.2292	-5.2292	-5.4332	-5.4332	-5.6337	-5.6337	-5.8342	-5.8342	-6.0347	-6.0347							
2			-6.9000			-6.8183	-6.8183	-6.7214	-6.7214	-6.6119	-6.6119	-6.5016	-6.5016	-6.3912	-6.3912	-6.2808	-6.2808							
1			-6.9001			-6.8184	-6.8184	-6.7215	-6.7215	-6.6121	-6.6121	-6.5018	-6.5018	-6.3914	-6.3914	-6.2810	-6.2810							

TABLE NO. 3.17:-

EIGENVECTORS (Z-TRANSLATION ONLY) FOR DIFFERENT COMBINATION
OF PLATE THICKNESS AND ANCHOR DIAMETERS.

$K_c = 2.0E+08 \text{ KN/m}^2\text{-m}$
Mesh size = 50X25
Model type I

node num	Thickness of plate in mm											
	16		20		24		28		32		40	
	Anchor diameter in mm											
	32	25	32	25	32	25	32	25	32	25	32	25
36							4.3916	4.3916	1.9694	1.9694		
35							1.5039	1.5039	-.2280	-.2280		
34							-.2134	-.2134	-1.5385	-1.5385		
33							-.6234	-.6234	-1.6710	-1.6710		
32							2.9998	2.9998	.9367	.9367		
31							-.0806	-.0806	-1.4673	-1.4673		
30							-2.0265	-2.0265	-3.0109	-3.0109		
29							-2.4916	-2.4916	-3.3934	-3.3934		
28							1.7151	1.7151	.0127	.0127		
27							-1.6499	-1.6499	-2.6916	-2.6916		
26							-3.9486	-3.9486	-4.5249	-4.5249		
25							-4.3807	-4.3807	-4.8799	-4.8799		
24							.6619	.6619	-.7134	-.7134		
23							-3.0236	-3.0236	-3.7508	-3.7508		
22							-5.7897	-5.7897	-5.9072	-5.9072		
21							-6.0103	-6.0103	-6.0869	-6.0869		
20							-.0585	-.0585	-1.1952	-1.1952		
19							-3.9532	-3.9532	-4.4598	-4.4598		
18							-6.7592	-6.7592	-6.6072	-6.6072		
17							-6.7619	-6.7619	-6.6109	-6.6109		
16							-.3927	-.3927	-1.4162	-1.4162		
15							-4.3419	-4.3419	-4.7607	-4.7607		
14							-6.7676	-6.7676	-6.6178	-6.6178		
13							-6.7691	-6.7691	-6.6199	-6.6199		
12							-.4778	-.4778	-1.4707	-1.4707		
11							-4.4233	-4.4233	-4.8298	-4.8298		
10							-6.7742	-6.7742	-6.6261	-6.6261		
9							-6.7750	-6.7750	-6.6272	-6.6272		
8							-.4603	-.4603	-1.4555	-1.4555		
7							-4.4119	-4.4119	-4.8263	-4.8263		
6							-6.7784	-6.7784	-6.6313	-6.6313		
5							-6.7788	-6.7788	-6.6320	-6.6320		
4							-.4426	-.4426	-1.4418	-1.4418		
3							-4.3995	-4.3995	-4.8188	-4.8188		
2							-6.7798	-6.7798	-6.6331	-6.6331		
1							-6.7801	-6.7801	-6.6336	-6.6336		

TABLE NO 3 19 -

EIGENVECTORS (Z-TRANSLATION ONLY) FOR DIFFERENT COMBINATION
OF PLATE THICKNESS AND ANCHOR DIAMETERS.

Kc = 4.0E+06 KN/m²/m
Mesh size = 50X25
Model type II

node num	Thickness of plate in mm											
	16	20	24	28	32	40						
Anchor diameter in mm												
	32	25	32	25	32	25	32	25	32	25	32	25
36		3069	- .7291	-1.4370	-2.3088	-2.7786	-3.4648	4.9203	-4.2328	5.4045	5.7143	
35	-1.8980	-2.6138	-3.1141	-3.7089	-4.0001	-4.4628	5.3576	-4.9407	5.6188	5.8227		
34	-3.6358	-4.0252	-4.4342	-4.7572	-4.9625	-5.2110	5.7613	-5.4722	5.8198	5.9246		
33	-4.2575	-4.5093	-4.9088	-5.1189	-5.3097	-5.4702	5.9245	-5.6570	5.9018	5.9663		
32	- .5521	-1.4768	-2.0947	-2.8649	-3.2621	-3.8622	5.1002	-4.5151	5.4934	5.7595		
31	-2.7590	-3.3768	-3.7477	-4.2523	-4.4545	-4.8420	5.5112	-5.2062	5.6939	5.8609		
30	-4.5267	-4.8129	-5.0635	-5.2954	-5.4028	-5.5779	5.9044	-5.7254	5.8894	5.9601		
29	-5.1025	-5.2630	-5.4952	-5.6249	-5.7156	-5.8116	6.0493	-5.8910	5.9622	5.9971		
28	-1.4095	-2.2119	-2.7559	-3.4151	-3.7497	-4.2567	5.2919	-4.7959	5.5886	5.8080		
27	-3.6590	-4.1595	-4.4069	-4.8090	-4.9282	-5.2303	5.6809	-5.4780	5.7773	5.9033		
26	-5.4690	-5.6338	-5.7278	-5.8552	-5.8670	-5.9592	6.0626	-5.9885	5.9688	5.9995		
25	-5.9264	-5.9924	-6.0656	-6.1132	-6.1098	-6.1405	6.1741	-6.1163	6.0228	6.0279		
24	-2.2322	-2.8973	-3.3952	-3.9315	-4.2232	-4.6285	5.4962	-5.0613	5.6909	5.8601		
23	-4.5666	-4.9125	-5.0733	-5.3429	-5.4038	-5.6026	5.8768	-5.7388	5.8750	5.9531		
22	-6.3437	-6.3816	-6.3410	-6.3622	-6.2943	-6.3036	6.2170	-6.2257	6.0428	6.0381		
21	-6.5484	-6.5430	-6.4908	-6.4771	-6.4015	-6.3839	6.2656	-6.2822	6.0673	6.0506		
20	-2.8952	-3.4443	-3.9206	-4.3502	-4.6162	-4.9325	5.6763	-5.2795	5.7817	5.9064		
19	-5.2422	-5.4624	-5.5721	-5.7356	-5.7628	-5.8776	6.0350	-5.9322	5.9544	5.9935		
18	-6.7869	-6.7559	-6.6509	-6.6152	-6.5101	-6.4753	6.2982	-6.3441	6.0830	6.0586		
17	-6.7873	-6.7562	-6.6513	-6.6156	-6.5106	-6.4756	6.2985	-6.3444	6.0832	6.0587		
16	-3.2864	-3.7770	-4.2496	-4.6166	-4.8694	-5.1303	5.7959	-5.4232	5.8425	5.9375		
15	-5.5594	-5.7257	-5.8165	-5.9303	-5.9428	-6.0166	6.1165	-6.0310	5.9957	6.0146		
14	-6.7876	-6.7565	-6.6517	-6.6159	-6.5110	-6.4760	6.2987	-6.3448	6.0834	6.0588		
13	-6.7878	-6.7566	-6.6519	-6.6161	-6.5113	-6.4762	6.2988	-6.3450	6.0836	6.0589		
12	-3.4319	-3.9231	-4.3974	-4.7478	-4.9922	-5.2325	5.8525	-5.4993	5.8717	5.9526		
11	-5.6187	-5.7929	-5.8779	-5.9889	-5.9941	-6.0617	6.1372	-6.0644	6.0066	6.0203		
10	-6.7881	-6.7569	-6.6523	-6.6165	-6.5117	-6.4766	6.2991	-6.3454	6.0838	6.0590		
9	-6.7883	-6.7570	-6.6525	-6.6166	-6.5118	-6.4767	6.2991	-6.3455	6.0838	6.0590		
8	-3.4282	-3.9525	-4.4271	-4.7892	-5.0265	-5.2692	5.8647	-5.5282	5.8783	5.9561		
7	-5.5455	-5.7658	-5.8445	-5.9810	-5.9780	-6.0603	6.1223	-6.0651	5.9994	6.0167		
6	-6.7884	-6.7572	-6.6527	-6.6168	-6.5121	-6.4769	6.2993	-6.3458	6.0840	6.0591		
5	-6.7885	-6.7573	-6.6528	-6.6169	-6.5122	-6.4770	6.2993	-6.3458	6.0840	6.0591		
4	-3.3995	-3.9460	-4.4195	-4.7927	-5.0264	-5.2747	5.8617	-5.5332	5.8770	5.9556		
3	-5.4750	-5.7312	-5.8046	-5.9626	-5.9538	-6.0497	6.1051	-6.0584	5.9908	6.0125		
2	-6.7886	-6.7573	-6.6528	-6.6169	-6.5123	-6.4770	6.2993	-6.3459	6.0840	6.0591		
1	-6.7886	-6.7574	-6.6529	-6.6170	-6.5124	-6.4771	6.2994	-6.3459	6.0841	6.0592		

TABLE NO. 320 -

EIGENVECTORS (Z-TRANSLATION ONLY) FOR DIFFERENT COMBINATION
OF PLATE THICKNESS AND ANCHOR DIAMETERS.

$K_c = 2.0E+07 \text{ KN/m}^2/\text{m}$
Mesh size = 50X25
Model type II

node num	Thickness of plate in mm											
	16		20		24		28		32		40	
	Anchor diameter in mm											
	32	25	32	25	32	25	32	25	32	25	32	25
36			2.8666	2.3376	1.2448	4.7655	-0.0809	-0.3698	4.9203	-1.3652	-2.7900	-2.9891
35			-0.0573	-0.3839	-1.2605	5.3146	-2.2428	-2.4570	5.3576	-3.1569	-4.0431	-4.1789
34			-2.2463	-2.3901	-3.1760	5.8153	-3.8919	-4.0120	5.7613	-4.4827	-4.9837	-5.0551
33			-3.1144	-3.1870	-3.9268	6.0160	-4.5267	-4.6045	5.9245	-4.9795	-5.3347	-5.3788
32			1.7700	1.3002	0.3430	4.9913	-0.8217	-1.0853	5.1002	-1.9793	-3.2242	-3.3989
31			-1.2434	-1.5391	-2.2316	5.5094	-3.0221	-3.2123	5.5112	-3.7712	-4.4515	-4.5660
30			-3.5393	-3.6543	-4.1935	5.9977	-4.6763	-4.7657	5.9044	-5.0675	-5.3539	-5.4038
29			-4.3584	-4.4076	-4.8815	6.1761	-5.2468	-5.2960	6.0493	-5.5047	-5.6586	-5.6838
28			0.7225	0.3274	-0.5197	5.2308	-1.5453	-1.7783	5.2919	-2.5795	-3.6554	-3.8031
27			-2.4613	-2.7101	-3.2249	5.7234	-3.8217	-3.9797	5.6809	-4.3956	-4.8698	-4.9595
26			-4.9057	-4.9790	-5.2620	6.1985	-5.4997	-5.5515	6.0626	-5.6768	-5.7417	-5.7669
25			-5.5780	-5.5994	-5.8108	6.3358	-5.9457	-5.9647	6.1741	-6.0117	-5.9718	-5.9776
24			-0.2042	-0.5082	-1.2962	5.4840	-2.2152	-2.4090	5.4962	-3.1311	-4.0611	-4.1783
23			-3.6197	-3.7906	-4.1659	5.9675	-4.5810	-4.6893	5.8768	-4.9722	-5.2649	-5.3228
22			-6.1929	-6.2150	-6.2526	6.3935	-6.2577	-6.2679	6.2170	-6.2289	-6.0937	-6.0937
21			-6.5044	-6.5040	-6.5017	6.4533	-6.4574	-6.4529	6.2656	-6.3772	-6.1946	-6.1861
20			-0.8957	-1.1274	-1.9041	5.7060	-2.7592	-2.9188	5.6763	-3.5832	-4.4002	-4.4903
19			-4.4531	-4.5587	-4.8500	6.1631	-5.1400	-5.2063	6.0350	-5.3953	-5.5602	-5.5918
18			-6.8600	-6.8522	-6.7596	6.4955	-6.6439	-6.6309	6.2982	-6.5077	-6.2722	-6.2585
17			-6.8607	-6.8528	-6.7604	6.4956	-6.6449	-6.6318	6.2985	-6.5087	-6.2734	-6.2597
16			-1.2719	-1.4768	-2.2728	5.8529	-3.1128	-3.2571	5.7959	-3.8924	-4.6366	-4.7101
15			-4.8317	-4.9143	-5.1781	6.2635	-5.4206	-5.4695	6.1165	-5.6169	-5.7186	-5.7374
14			-6.8612	-6.8533	-6.7611	6.4957	-6.6457	-6.6326	6.2987	-6.5096	-6.2744	-6.2606
13			-6.8616	-6.8537	-6.7616	6.4958	-6.6463	-6.6331	6.2988	-6.5102	-6.2752	-6.2613
12			-1.4130	-1.6367	-2.4530	5.9221	-3.3082	-3.4578	5.8525	-4.0828	-4.7821	-4.8501
11			-4.9161	-5.0166	-5.2742	6.2889	-5.5169	-5.5706	6.1372	-5.7080	-5.7843	-5.8016
10			-6.8622	-6.8543	-6.7622	6.4959	-6.6471	-6.6339	6.2991	-6.5111	-6.2762	-6.2623
9			-6.8624	-6.8545	-6.7625	6.4960	-6.6474	-6.6342	6.2991	-6.5114	-6.2766	-6.2627
8			-1.4340	-1.6996	-2.5227	5.9369	-3.4004	-3.5661	5.8647	-4.1883	-4.8598	-4.9294
7			-4.8776	-5.0241	-5.2697	6.2706	-5.5299	-5.6021	6.1223	-5.7404	-5.8045	-5.8270
6			-6.8627	-6.8548	-6.7630	6.4960	-6.6479	-6.6347	6.2993	-6.5120	-6.2773	-6.2633
5			-6.8629	-6.8550	-6.7632	6.4961	-6.6482	-6.6349	6.2993	-6.5122	-6.2776	-6.2636
4			-1.4306	-1.7221	-2.5426	5.9333	-3.4304	-3.6062	5.8617	-4.2268	-4.8861	-4.9580
3			-4.8348	-5.0153	-5.2490	6.2497	-5.5212	-5.6075	6.1051	-5.7474	-5.8063	-5.8334
2			-6.8629	-6.8550	-6.7632	6.4961	-6.6482	-6.6350	6.2993	-6.5123	-6.2777	-6.2637
1			-6.8631	-6.8552	-6.7634	6.4961	-6.6484	-6.6352	6.2994	-6.5125	-6.2779	-6.2639

TABLE NO. 3.21

EIGENVECTORS (Z-TRANSLATION ONLY) FOR DIFFERENT COMBINATION
OF PLATE THICKNESS AND ANCHOR DIAMETERS.

$K_c = 4.0E+07 \text{ KN/m}^2/\text{m}$
Mesh size = 50×25
Model type II

node num	Thickness of plate in mm											
	16		20		24		28		32		40	
	Anchor diameter in mm											
	32	25	32	25	32	25	32	25	32	25	32	25
36		4.5178		2.7388	1.9959	1.4217	.8788	.3179	.1130	-1.3596	-1.5001	
35		1.5580		.2236	-.2406	-.8108	-1.1715	-1.6883	-1.8321	-2.9647	-3.0639	
34		-.3221		-1.3885	-1.5861	-2.2971	-2.4716	-3.0542	-3.1293	-4.0738	-4.1263	
33		-.8261		-1.8456	-1.9350	-2.7414	-2.8361	-3.4754	-3.5201	-4.4267	-4.4586	
32		3.1151		1.5879	.9457	.4448	-.0250	-.5118	-.6903	-1.9973	-2.1227	
31		-.0283		-1.1060	-1.5123	-1.9402	-2.2565	-2.6330	-2.7580	-3.6302	-3.7159	
30		-2.1458		-2.9451	-3.1006	-3.5893	-3.7230	-4.1028	-4.1588	-4.7588	-4.7966	
29		-2.6975		-3.4404	-3.5007	-4.0486	-4.1104	-4.5213	-4.5496	-5.0893	-5.1081	
28		1.8077		.5249	.0032	-.4576	-.8441	-1.2914	-1.4419	-2.6094	-2.7175	
27		-1.6155		-2.4387	-2.7694	-3.0880	-3.3262	-3.5773	-3.6788	-4.2960	-4.3651	
26		-4.0809		-4.5638	-4.6814	-4.9188	-4.9995	-5.1772	-5.2098	-5.4583	-5.4786	
25		-4.5811		-5.0069	-5.0322	-5.3160	-5.3408	-5.5283	-5.5389	-5.7228	-5.7280	
24		.7074		-.3552	-.7365	-1.2181	-1.5118	-1.9659	-2.0852	-3.1534	-3.2405	
23		-3.0503		-3.6395	-3.8584	-4.0802	-4.2523	-4.4261	-4.4939	-4.8949	-4.9406	
22		-5.9317		-6.0616	-6.0907	-6.1250	-6.1452	-6.1415	-6.1480	-6.0785	-6.0801	
21		-6.1801		-6.2784	-6.2767	-6.3152	-6.3111	-6.3065	-6.3035	-6.1995	-6.1944	
20		-.0569		-.9604	-1.2281	-1.7597	-1.9825	-2.4695	-2.5627	-3.5715	-3.6449	
19		-4.0335		-4.4609	-4.5865	-4.7770	-4.8773	-5.0168	-5.0564	-5.3185	-5.3445	
18		-6.8996		-6.8229	-6.8124	-6.7286	-6.7144	-6.6205	-6.6126	-6.3849	-6.3762	
17		-6.9003		-6.8238	-6.8132	-6.7297	-6.7155	-6.6219	-6.6139	-6.3866	-6.3778	
16		-.4002		-1.2403	-1.4533	-2.0393	-2.2277	-2.7526	-2.8331	-3.8390	-3.8967	
15		-4.4334		-4.8047	-4.8924	-5.0819	-5.1526	-5.2869	-5.3146	-5.5253	-5.5421	
14		-6.9016		-6.8254	-6.8148	-6.7315	-6.7173	-6.6239	-6.6159	-6.3889	-6.3801	
13		-6.9019		-6.8259	-6.8153	-6.7322	-6.7179	-6.6247	-6.6166	-6.3899	-6.3810	
12		-.4494		-1.2933	-1.5089	-2.1259	-2.3183	-2.8670	-2.9484	-3.9711	-4.0267	
11		-4.4766		-4.8583	-4.9597	-5.1467	-5.2259	-5.3578	-5.3879	-5.5942	-5.6109	
10		-6.9031		-6.8273	-6.8167	-6.7338	-6.7195	-6.6265	-6.6185	-6.3920	-6.3831	
9		-6.9033		-6.8276	-6.8169	-6.7342	-6.7198	-6.6270	-6.6189	-6.3926	-6.3836	
8		-.3725		-1.2455	-1.4944	-2.1164	-2.3333	-2.8877	-2.9775	-4.0201	-4.0788	
7		-4.3869		-4.8043	-4.9545	-5.1198	-5.2321	-5.3493	-5.3908	-5.6036	-5.6259	
6		-6.9040		-6.8285	-6.8179	-6.7353	-6.7209	-6.6282	-6.6201	-6.3940	-6.3850	
5		-6.9042		-6.8287	-6.8180	-6.7355	-6.7211	-6.6285	-6.6203	-6.3943	-6.3853	
4		-.3202		-1.2103	-1.4815	-2.0987	-2.3314	-2.8834	-2.9788	-4.0296	-4.0909	
3		-4.3172		-4.7592	-4.9467	-5.0914	-5.2292	-5.3322	-5.3825	-5.5983	-5.6253	
2		-6.9044		-6.8289	-6.8183	-6.7358	-6.7214	-6.6287	-6.6208	-6.3946	-6.3857	
1		-6.9045		-6.8291	-6.8184	-6.7360	-6.7215	-6.6290	-6.6208	-6.3949	-6.3859	

TABLE NO 3 22 -

EIGENVECTORS (Z-TRANSLATION ONLY) FOR DIFFERENT COMBINATION
OF PLATE THICKNESS AND ANCHOR DIAMETERS.

$K_c = 2.0E+07 \text{ KN/m}^2/\text{m}$
Mesh size = 50X25
Model type II

node num	Thickness of plate in mm																	
	16			20			24			28			32			40		
	Anchor diameter in mm																	
	32	25	32	25	32	25	32	25	32	25	32	25	32	25				
36					10.6800	10.9200	6.7363	6.6011	4.5086	4.3953	2.2180	2.1439						
35					4.7788	4.9388	2.6247	2.5624	1.2731	1.2110	- .3856	- .4297						
34					.7304	.8093	- .2828	- .2913	-1.0116	-1.0299	-2.2709	-2.2884						
33					- .6840	- .6367	-1.3846	-1.3763	-1.9289	-1.9318	-3.0412	-3.0484						
32					8.7835	8.9715	5.1732	5.0463	3.1980	3.0979	1.2252	1.1598						
31					2.7976	2.9215	.9601	.9030	- .1719	- .2266	-1.5088	-1.5469						
30					-1.2032	-1.1485	-2.0117	-2.0199	-2.5916	-2.6069	-3.5007	-3.5149						
29					-2.5972	-2.5666	-3.1099	-3.1041	-3.4983	-3.5002	-4.2291	-4.2337						
28					6.9636	7.0951	3.6788	3.5612	1.9656	1.8815	.3068	.2527						
27					.7730	.8589	- .7211	- .7706	-1.6201	-1.6654	-2.6360	-2.6698						
26					-3.3455	-3.3133	-3.8920	-3.8977	-4.2673	-4.2773	-4.7811	-4.7900						
25					-4.5531	-4.5374	-4.8474	-4.8435	-5.0500	-5.0502	-5.3840	-5.3854						
24					5.3878	5.4569	2.3616	2.2523	.9091	.8446	- .4599	- .5000						
23					-1.0369	- .9917	-2.2433	-2.2833	-2.9271	-2.9584	-3.6483	-3.6708						
22					-5.5432	-5.5297	-5.7647	-5.7669	-5.8801	-5.8834	-5.9702	-5.9723						
21					-6.1375	-6.1307	-6.2340	-6.2324	-6.2616	-6.2608	-6.2554	-6.2543						
20					4.1964	4.2026	1.3288	1.2225	.1282	.0822	-1.0202	-1.0490						
19					-2.2991	-2.2879	-3.3321	-3.3662	-3.8508	-3.8699	-4.3618	-4.3748						
18					-6.7904	-6.7862	-6.7967	-6.7969	-6.7428	-6.7421	-6.5871	-6.5852						
17					-6.7931	-6.7888	-6.7998	-6.8000	-6.7464	-6.7456	-6.5919	-6.5899						
16					3.4887	3.4362	.6758	.5638	- .3043	- .3389	-1.3456	-1.3695						
15					-2.8677	-2.8819	-3.8513	-3.8868	-4.2783	-4.2926	-4.7017	-4.7110						
14					-6.7954	-6.7912	-6.8029	-6.8031	-6.7500	-6.7493	-6.5963	-6.5944						
13					-6.7971	-6.7928	-6.8048	-6.8050	-6.7522	-6.7515	-6.5992	-6.5972						
12					3.1210	3.0160	.3065	.1810	- .5058	- .5396	-1.5210	-1.5465						
11					-3.0512	-3.0887	-4.0423	-4.0875	-4.4210	-4.4387	-4.8247	-4.8359						
10					-6.7994	-6.7952	-6.8078	-6.8081	-6.7558	-6.7551	-6.6036	-6.6016						
9					-6.8005	-6.7963	-6.8090	-6.8093	-6.7572	-6.7564	-6.6054	-6.6034						
8					2.9494	2.8039	.1191	- .0221	- .5987	- .6390	-1.6219	-1.6524						
7					-3.1334	-3.1963	-4.1346	-4.1961	-4.4813	-4.5089	-4.8777	-4.8948						
6					-6.8021	-6.7979	-6.8110	-6.8114	-6.7595	-6.7588	-6.6083	-6.6063						
5					-6.8029	-6.7987	-6.8119	-6.8122	-6.7605	-6.7597	-6.6095	-6.6075						
4					2.8927	2.7301	.0561	- .0939	- .6378	- .6832	-1.6684	-1.7022						
3					-3.1794	-3.2580	-4.1808	-4.2539	-4.5123	-4.5476	-4.9015	-4.9230						
2					-6.8030	-6.7988	-6.8122	-6.8125	-6.7608	-6.7602	-6.6100	-6.6080						
1					-6.8037	-6.7995	-6.8129	-6.8132	-6.7616	-6.7609	-6.6110	-6.6090						

TABLE NO. 3 E31-

EIGENVECTOS (Z-TRANSLATION ONLY) FOR DIFFERENT COMBINATION
OF PLATE THICKNESS AND ANCHOR DIAMETERS.

$K_c = 4.0E+08 \text{ (KN/m}^2\text{)m}$
Mesh size = 50X25
Model type II

node num	Thickness of plate in mm											
	16		20		24		28		32		40	
	Anchor diameter in mm											
	32	25	32	25	32	25	32	25	32	25	32	25
36						4.7655						
35						5.3146						
34						5.8153						
33						6.0160						
32						4.9913						
31						5.5094						
30						5.9977						
29						6.1761						
28						5.2308						
27						5.7234						
26						6.1985						
25						6.3358						
24						5.4840						
23						5.9675						
22						6.3935						
21						6.4533						
20						5.7060						
19						6.1631						
18						6.4955						
17						6.4956						
16						5.8529						
15						6.2635						
14						6.4957						
13						6.4958						
12						5.9221						
11						6.2889						
10						6.4959						
9						6.4960						
8						5.9369						
7						6.2706						
6						6.4960						
5						6.4961						
4						5.9333						
3						6.2497						
2						6.4961						
1						6.4961						

TABLE 3.24 -

FREQUENCY OF THE EMBEDDED PLATE ASSEMBLY FOR MODEL TYPE I
AND ANCHOR DIAMETER 32 mm

Thickness of the plate (mm)	Frequency (rad./sec)									
	Base stiffness (kn/m ² /m)									
	4x10 ⁶		2x10 ⁷		4x10 ⁷		2x10 ⁸		4x10 ⁸	
	Mesh size (mm)									
	50x50	50x25	50x50	50x25	50x50	50x25	50x50	50x25	50x50	50x25
16	2180	A	4134	A	5496	A	B	A	B	A
20	2296	2261	4323	B	5794	5731	B	B	B	B
24	2362	2317	4598	4571	6062	6002	B	B	B	B
28	2391	2337	B	B	6300	6235	B	B	B	B
32	2397	2048	2076	2084	6498	6422	12470	12410	B	B
40	2048	2058	5013	4935	6761	6659	13070	13010	B	B
50									18120	17940
60									18670	18460
80									19180	18880

NOTE:- A : Rigid body mode was found .

B : No convergence took place .

TABLE 3.25 -

FREQUENCY OF THE EMBEDDED PLATE ASSEMBLY FOR MODEL TYPE II
AND ANCHOR DIAMETER 32 mm.

Thickness of the plate (mm)	Frequency (Yad /sec)									
	Base stiffness (kn/m ² /m)									
	4x10 ⁶		2x10 ⁷		4x10 ⁷		2x10 ⁸		4x10 ⁸	
	Mesh size (mm)									
	50x50	50x25	50x50	50x25	50x50	50x25	50x50	50x25	50x50	50x25
16	2590	A	4251	A	5543	A	B	A	B	A
20	2820	2799	B	3920	5853	5822	B	B	B	B
24	2966	2942	4811	4188	6184	6124	B	9486	B	B
28	3050	3020	5015	4394	6446	6384	B	9977	B	B
32	3089	3054	2076	2084	6665	6594	12510	10380	B	B
40	3091	3050	5325	4706	6959	6865	13130	11060	B	B
50									18160	17980
60									18720	18510
80									19240	18940

NOTE - A : Rigid body mode was found .

B : No convergence took place .

TABLE 3.26 -

FREQUENCY OF THE EMBEDDED PLATE ASSEMBLY FOR MODEL TYPE I
AND ANCHOR DIAMETER 25 mm.

Thickness of the plate (mm)	Frequency (rad/sec)									
	Base stiffness (kn/m ² /m)									
	4x10 ⁶		2x10 ⁷		4x10 ⁷		2x10 ⁸		4x10 ⁸	
	Mesh size (mm)									
	50x50	50x25	50x50	50x25	50x50	50x25	50x50	50x25	50x50	50x25
16	2180	A	4134	A	5496	A	B	A	B	A
20	2296	2261	4383	B	B	B	B	B	B	B
24	2362	2317	1501	4571	6062	6002	1501	B	B	1507
28	2391	2337	1501	1507	6300	6235	B	B	B	B
32	2397	1497	4888	4838	6498	6422	12470	12410	1490	B
40	1459	1466	5013	4935	6662	6659	13070	13010	B	B
50									18120	17940
60									18670	18460
80									19180	18880

NOTE - A : Rigid body mode was found .

B : No convergence took place .

TABLE 3.27 :-

FREQUENCY OF THE EMBEDDED PLATE ASSEMBLY FOR MODEL TYPE II
AND ANCHOR DIAMETER 25 mm

Thickness of the plate (mm)	Frequency (rad/sec)									
	Base stiffness (kn/m ² /m)									
	4x10 ⁶		2x10 ⁷		4x10 ⁷		2x10 ⁸		4x10 ⁸	
	Mesh size (mm)									
	50x50	50x25	50x50	50x25	50x50	50x25	50x50	50x25	50x50	50x25
16	2421	A	4198	A	5522	A	B	A	B	A
20	2593	2569	B	3823	B	B	B	B	B	B
24	2699	2667	1501	1507	6126	6066	1501	9474	9269	B
28	2753	2715	1501	4252	6375	6312	B	9958	B	B
32	2774	2730	5030	4388	6583	6510	12490	10350	1490	B
40	1459	1466	5173	4528	6862	6764	13100	11020	B	B
50									18140	17960
60									18700	18490
80									19210	18910

NOTE :- A : Rigid body mode was found .

B : No convergence took place .

CHAPTER 4

SUMMARY AND CONCLUSIONS

4.1 SUMMARY :-

Embedded plate-anchor assembly is very common in modern civil engineering constructions where it may be subjected to static and dynamic loads arising due to dead load ,machine vibrations, seismic inertias ,and fluid dynamic forces or combination thereof .

The need of conducting a free vibration analysis is described in section 1.1. The free-vibration analysis of any system is an essential first step towards obtaining its solution for the forced vibration analysis .

Section 1.2 discusses about the embedded plate-anchor system . The plate-anchor assembly consists of a steel plate embedded in concrete base and attached to the base with anchors at certain locations ,as shown in Fig. 1.1 .

A brief literature survey is given in section 1.3 .Though no work has been found to deal with problem directly ,but provides certain background to it .Section 1.4 gives outlines of the object and scope of the investigation .

Chapter 2 deals with the developement of the plate-anchor assembly model and the method of analysis .The assumptions made on the behaviour of the plate , the anchor ,and the concrete base are given in section 2.1 .Different components are discussed separately . Section 2.2 discusses about the different models and the one which is chosen for the present study with the assumptions made .

The finite element technique as employed in the present work using SAP IV computer programme ,is discussed in section 2.6. An idealized finite element model was prepared for the plate-anchor assembly . Symmetry was exploited in the free vibration analysis of the system ,to reduce the problem size .Determinant search technique was used in the computation of fundamental frequencies and eigenvectors .

An iterative scheme for determination of fundamental mode and frequency has been discussed in section 2.7 . Iterative schemes are illustarated in Fig.2.5 and Fig.2.6 for the two models adopted .The criterion for selecting various values of plate thickness, anchor diameters and base stiffness are discussed in section 2.8 .

The results obtained by finite element technique using SAP IV programme are presented through graphs and tables in chapter 3. A total of 240 cases were studied and results are discussed in this chapter in length .

Based on the results obtained following conclusions are made regarding free vibration of the plate-anchor assembly embedded in concrete :

- 1> free vibration characteristics ,generally ,depend on the parametric combination of plate thickness ,anchor diameters ,and concrete base stiffness .
- 2> The magnitude of plate-assembly frequency has been found to be influenced by the idealised value of the concrete base stiffness .Generally ,the frequency increased by about 3-5 times when the base stiffness value was increased by about 50 times (i.e. from $4 \times 10^6 \text{ KN/m}^2/\text{m}$ {flexible base} to $2 \times 10^8 \text{ KN/m}^2/\text{m}$ {moderately rigid base}).
- 3> Refinement of finite element mesh in the analysis of plate assembly has marginal effect on the plate frequency when the anchor is considered inactive in compression . But,when the plate-assembly is modelled considering the anchor to be ctive both in tension and in compression ,frequency decreased to relatively lower values due to mesh refinement specially for the idealised moderately flexible base and moderately rigid base .
- 4> For the anchor sizes considered in this study,the idealised anchor-stiffness has got insignificantly small influence over the frequency of the plate-assembly .This is specifically true in case of embedded plate showing mode-shape pattern having base-contact over most of the plate area . Otherwise,frequency value decreases appreciably when the complete plate gets lifte-up (no

base contact) on lowering the anchor size .

- 5> For relatively high values of base-stiffness, (moderately rigid to rigid base) finite element analysis did not show convergence up to plate thickness of 40 mm. This trend was reversed when the plate thickness was increased beyond 40 mm. giving converged final frequencies and corresponding mode shapes .This clearly indicates that the idealisation of base-stiffness plays an important role in the free-vibration study of the plate-assembly .
- 6> Frequency of plate-assembly increases with the increase of the plate thickness .This trend is distinctly observed when the mode shape pattern has base-contact over most of the plate area .Reverse trend is true for cases with plate lift-off (no base contact) .

4.3. FURTHER SCOPE OF WORK :-

It has been thought that the following studies can be undertaken as extension of the present investigation :

- 1> Free-vibration study of embedded plate assembly for determination of higher modal frequencies and corresponding mode shapes .
- 2> Classical solutions for free-vibration problems of such plates (see Appendix A) for comparisons with the approximate numerical solutions (e.g. finite element technique)

APPENDIX - A

The typical embedded plate-anchor assembly as illustrated in Fig. 2.2 can be idealized as given in Fig. A 1.1 for the possible classical solution . Now the idealised model as given in Fig. A 1.1 can be viewed as a plate resulting due to the superimposition of two cases as illustrated in Fig. A 1.2. Proper care must be exercised regarding the continuity, compatibility, and boundary conditions of the problem while obtaining the final solutions through superimposition .

A 1.1 SOLUTION METHOD [15] FOR CASE -I : (See Figs. A 1.2 and A 1.3)

As solved by Gorman [15] pages 301-302 the governing differential equation is

$$\frac{\partial^4 W(x,y,t)}{\partial x^4} + 2 \frac{\partial^4 W(x,y,t)}{\partial x^2 \partial y^2} + \frac{\partial^4 W(x,y,t)}{\partial y^4} + D \frac{\rho}{\partial t^2} \frac{\partial^2 W(x,y,t)}{\partial t^2} + \frac{K}{D} W(x,y,t) = 0 \quad \text{-----} \quad (A1)$$

Expressing $W(x,y,t)$ as the product of two functions as

$$W(x,y,t) = W(x,y) \times T(t) \quad \text{-----} \quad (A2)$$

and on solving , circular frequency ω' is given as

$$\omega' = \sqrt{\omega^2 + K/\rho} \quad \text{-----} \quad (A3)$$

where ω' is a parameter appearing in eigenvalue formulation . According to Gorman [15] support given to rectangular plates by

homogeneous linear elastic foundation will in no way affect the eigenvalues or the mode shapes ,however the frequencies will be changed . Therefore the solution of the plate anchor assembly can be obtained from the solution of the plate resting on six point support with modified ω as discussed later .

A 1.2 SOLUTION METHOD [15] FOR CASE II : (See Figs A 1.2 and A 1.4)

Because of the symmetry only one quarter of the plate needs to be considered as shown in Fig. A 1.4. The pair of circles indicate that along these edges the plate has zero vertical edge reaction and that the slope of the plate taken normal to these edges is equal to zero .

The solution regarding mode shapes for above first three building blocks are obtained as described in Gorman [15] pages 260-268 . The solution regarding mode shape for fourth building block can be obtained as per the method given in pages 241-243 in the same book with certain modifications due to the locations of anchors as per the present case .

After obtaining the solution for above building blocks , the mode shape of the system can be written in the form based on the fact that the mode shape remains the same after superimposition of case I and case II .

$$W(x,y) = W_1(x,y) + W_2(x,y) + W_3(x,y) + W_{41}(x,y) + W_{42}(x,y)$$

- - - - - (A4)

Eigenvalue of the system can be obtained by imposing the

boundary and continuity conditions .In addition to continuity of displacement,slope,and bending moment the condition that the vertical edge reaction along the boundary of two segments just balances the applied force .

The solutions of the building blocks of second case are based on the following differential equation

$$\frac{\partial^4 W(x,y)}{\partial x^4} + \frac{\partial^4 W(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^4 W(x,y)}{\partial y^4} + \frac{\rho}{D} \frac{\partial^2 W(x,y)}{\partial t^2} = \omega^2 \frac{\rho}{D} W(x,y) \quad (A5)$$

Where, ω^2 ,when the plate is considered resting on elastic foundation and on point supports is given by

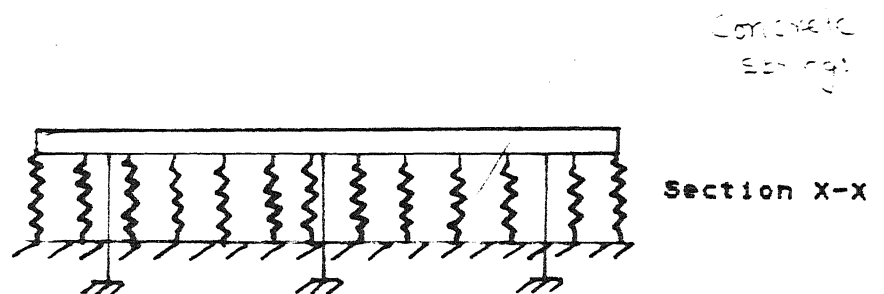
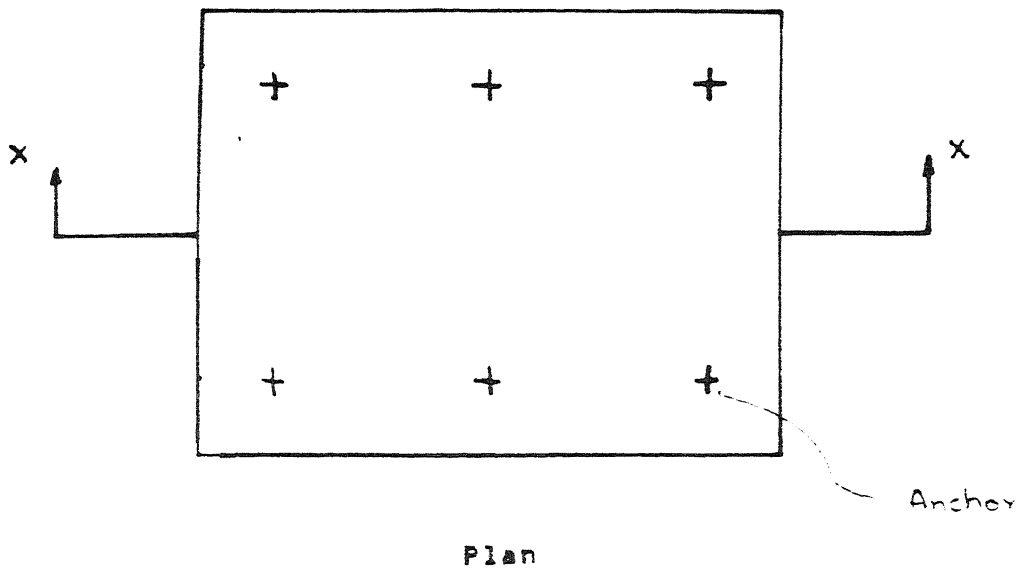
$$\omega^2 = -\frac{1}{T(t)} \frac{d^2 T(t)}{dt^2} - \frac{K}{\rho} \quad (A6)$$

$$\text{and } W(x,y,t) = W(x,y) \cdot T(t) \quad (A7)$$

$$\text{Eigenvalue } \lambda^2 = \omega a^2 \sqrt{\rho/D} \quad (A8)$$

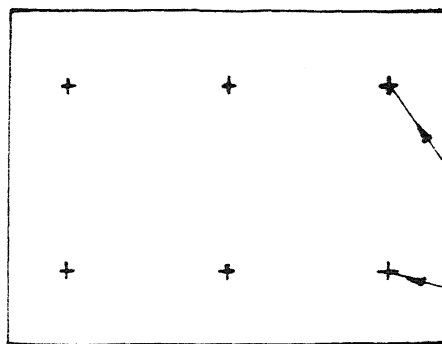
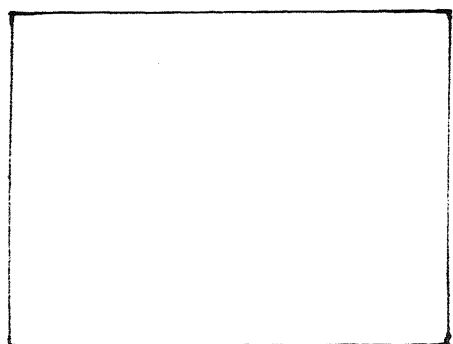
To start with all the concrete springs are considered to act . Also it is assumed that anchors act as rigid point support. From the eigenvalue ' λ^2 ', ω is calculated from (A8) and circular frequency of the system ω' is calculated from the equation (A2).mode shape is obtained from equation (A4). This equation of the mode shape is a function of X and Y co-ordinates. By selecting a grid of points the lift-off or compression of the plate can be determined.

Now using the finite difference method the differential equation as given by equation (A5) can be solved for the mode shape and fundamental frequency of the system satisfying the lift-off condition of the plate (i.e. no contact region of the plate). The procedure is repeated till all the active concrete springs come under compression.

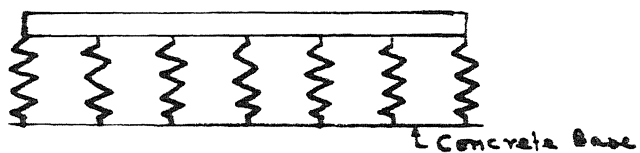


IDEALIZED REPRESENTATION OF THE PLATE ASSEMBLY MODEL .

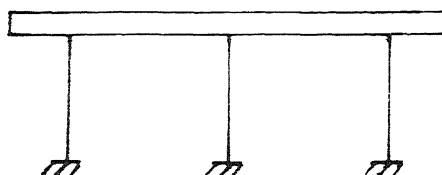
FIG. A 11



Anch



(a)

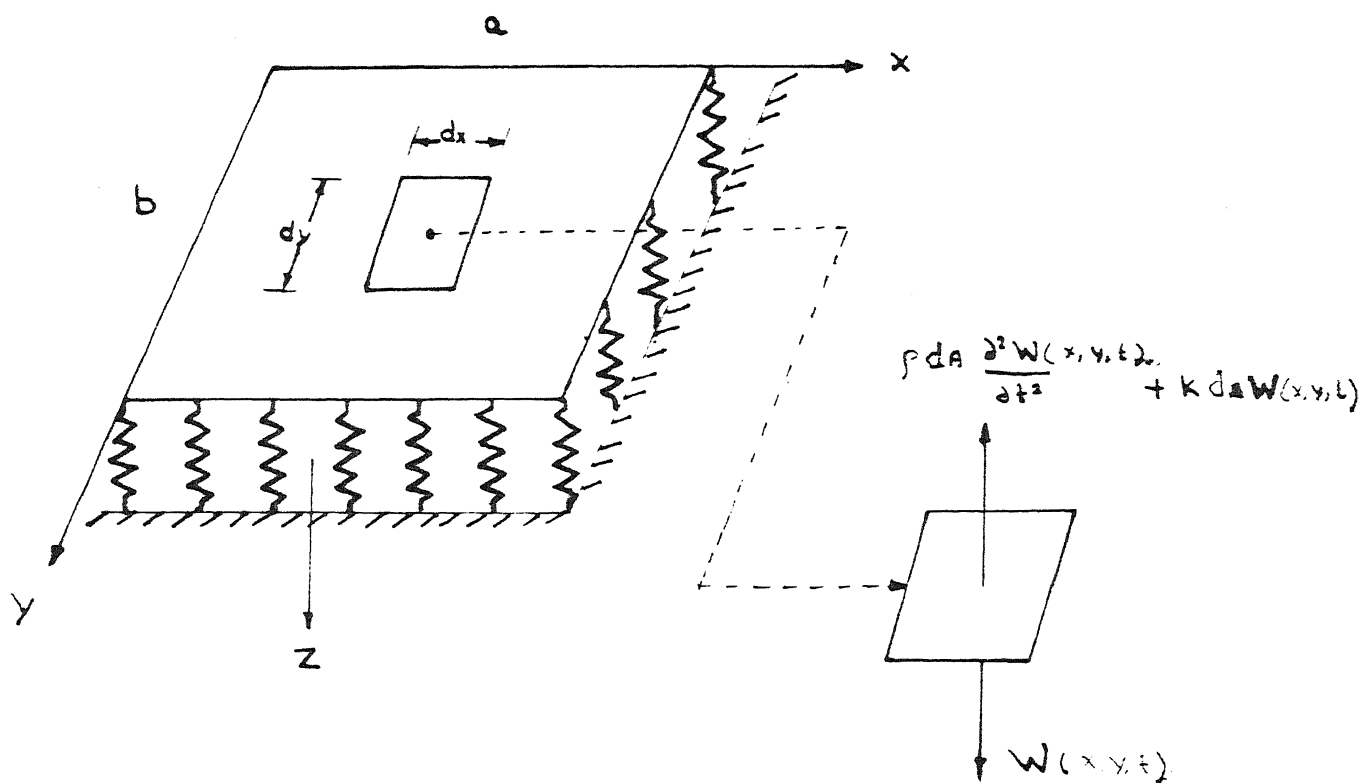


(b)

PLATE ASSEMBLY, COMBINATION OF TWO PROBLEMS .

FIG. A 12

A



RECTANGULAR PLATE RESTING ON ELASTIC FOUNDATION .

FIG. A 13

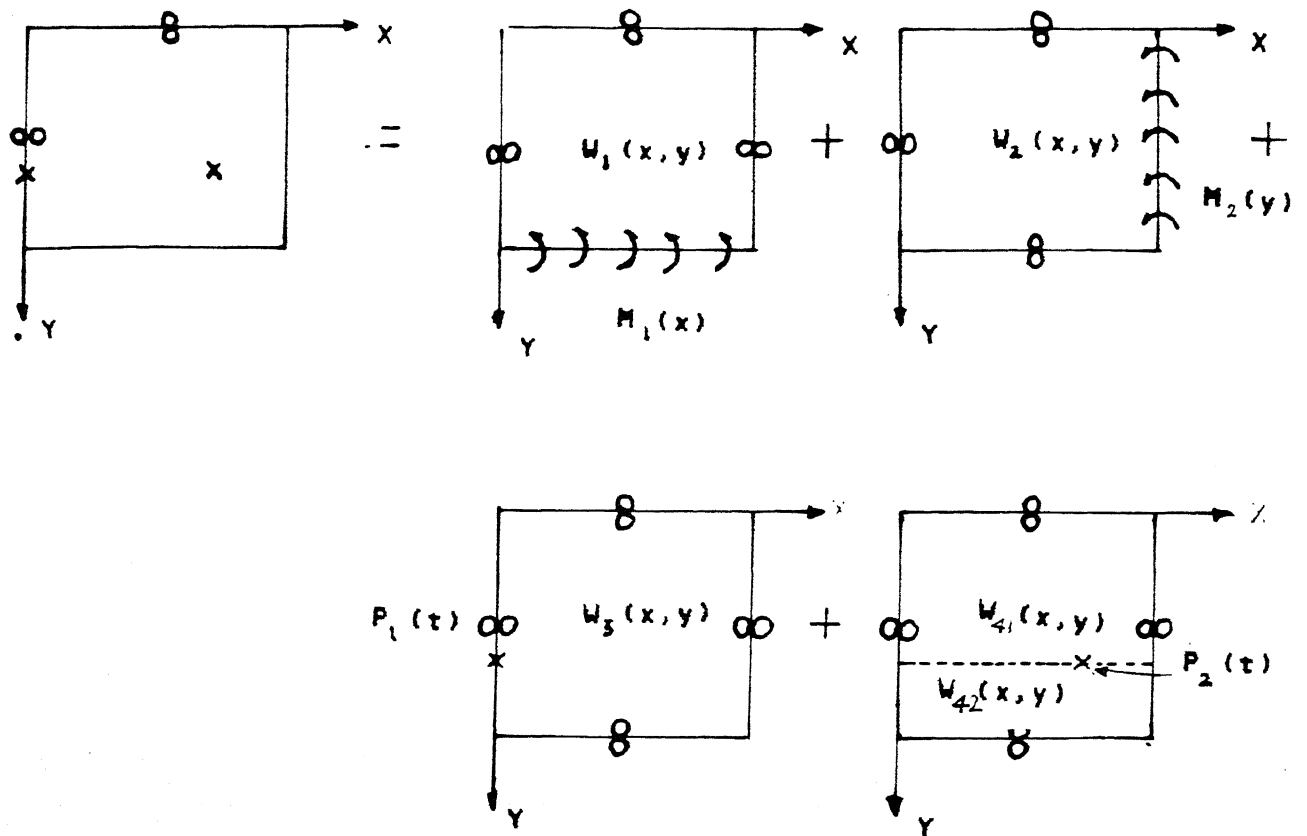


FIG. A 14

BUILDING BLOCKS FOR ANALYSING THE FULLY SYMMETRIC MODES OF RECTANGULAR PLATE WITH SYMMETRICALLY DISTRIBUTED POINT SUPPORT ON THE LATERAL SURFACE .

LIST OF REFERENCES :-

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- 2 Szillard ,Rudolph , "Theory and Analysis of Plates Classical and Numerical Methods ", Prentice-Hall ,Inc,Englewood Cliffs ,New Jersey .
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